OPTICAL FIBER SENSORS BASED ON FORWARD AND BACKWARD STIMULATED BRILLOUIN SCATTERING

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ASE</td>
<td>Amplified Spontaneous Emission</td>
</tr>
<tr>
<td>AWG</td>
<td>Arbitrary Waveform Generator</td>
</tr>
<tr>
<td>B-OCDA</td>
<td>Brillouin Correlation Time Domain Analysis</td>
</tr>
<tr>
<td>B-OTDA</td>
<td>Brillouin Optical Time Domain Analysis</td>
</tr>
<tr>
<td>BPF</td>
<td>Band-Pass Filter</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous wave</td>
</tr>
<tr>
<td>DBG</td>
<td>Dynamic Brillouin Grating</td>
</tr>
<tr>
<td>DFB</td>
<td>Distributed Feedback</td>
</tr>
<tr>
<td>DPP</td>
<td>Differential Pulse Pair</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium Doped Fiber Amplifier</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>EOM</td>
<td>Electro-Optic Modulator</td>
</tr>
<tr>
<td>FBG</td>
<td>Fiber Bragg Grating</td>
</tr>
<tr>
<td>FM</td>
<td>Frequency Modulation</td>
</tr>
<tr>
<td>FUT</td>
<td>Fiber Under Test</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width at Half Maximum</td>
</tr>
<tr>
<td>LF</td>
<td>Low Frequency</td>
</tr>
<tr>
<td>MFD</td>
<td>Mode Filed Diameter</td>
</tr>
<tr>
<td>NEP</td>
<td>Noise Equivalent Power</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>OFDR</td>
<td>Optical Frequency Domain Reflectometer</td>
</tr>
<tr>
<td>OOK</td>
<td>On-Off-Keying</td>
</tr>
<tr>
<td>OSNR</td>
<td>Optical Signal to Noise Ratio</td>
</tr>
<tr>
<td>OTDR</td>
<td>Optical Time Domain Reflectometer</td>
</tr>
<tr>
<td>PC</td>
<td>Polarization Controller</td>
</tr>
<tr>
<td>PCF</td>
<td>Photonic Crystal Fiber</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
</tr>
<tr>
<td>PM</td>
<td>Polarization Maintaining</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo Random Binary Sequence</td>
</tr>
<tr>
<td>RZ</td>
<td>Return-to-Zero</td>
</tr>
<tr>
<td>SBS</td>
<td>Stimulated Brillouin Scattering</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SOA</td>
<td>Semiconductor Optical Amplifier</td>
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<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
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Notation

\( \Gamma' \)  
Acoustic damping parameter

\( \Gamma_A \)  
Complex damping factor

\( \Gamma_B \)  
Brillouin linewidth

\( \Delta \phi \)  
Non-reciprocal phase

\( \Delta z \)  
Bin size / spatial resolution

\( \Lambda \)  
Grating period

\( \theta(z) \)  
Position-dependent temporal offset

\( \alpha \)  
Polarizability

\( \beta \)  
Propagation constant

\( \chi \)  
Electric susceptibility

\( \varepsilon \)  
Permittivity coefficient / strain

\( \phi_{\text{non-linear}} \)  
Nonlinear phase

\( \gamma \)  
Kerr nonlinearity coefficient

\( \gamma_e \)  
Electrostrictive constant

\( \mu_0 \)  
Vacuum permeability

\( \nu \)  
Frequency

\( \nu_B, \Omega_B \)  
Brillouin frequency shift in Hz / radians

\( \lambda \)  
Wavelength

\( \rho \)  
Mass density

\( \sigma^2_B \)  
Variance of off-peak correlation

\( \tau \)  
Acoustic life time

\( \omega, \Omega \)  
Angular frequency

\( A \)  
Complex amplitude of an electric field

\( A_{\text{eff}} \)  
Cross-section area

\( B \)  
Magnetic field

\( E \)  
Electric field / Young modulus

\( F_N \)  
Noise figure

\( G \)  
Gain

\( I \)  
Optical intensity / Moment of inertia

\( M_a \)  
Amplitude sequence length

\( M_p \)  
Phase sequence length
$N_0$  Exponential window width
$N_{\text{avg}}$  Number of averages
$N_{\text{bins}}$  Number of bins
$N_v$  Number of frequency samples
$L$  Fiber length / beam length
$P$  Optical power
$Q_e, Q_i$  Acousto-optic overlap integrals
$R$  Reflectivity
$\tilde{R}_{N_0}(l_z)$  Exponentially windowed correlation
$T$  Temperature
$T_a$  Amplitude symbol duration
$T_D$  Round-trip propagation time
$T_{\text{int}}$  Integration time
$a$  Cladding radius
$b$  Acoustic complex amplitude
$c$  Speed of light in vacuum
$c_n$  Phase code
$d_n$  Amplitude code
$f$  Electrostrictive force
$g$  SBS gain factor
$g_0$  Line-center gain factor
$h$  Planck constant
$k$  Wavenumber
$k_B$  Boltzmann constant
$n(x,y)$  Local refractive index
$n_{\text{eff}}$  Effective refractive index
$l_z$  Position-dependent lag
$p$  Dipole moment, momentum
$p_{\text{st}}$  Electrostrictive pressure
$q$  Electron charge / wavenumber
$r$  Radial position
$r_{\text{mir}}$  Acoustic reflectivity
\begin{align*}
s & \quad \text{Curve parameter} \\
t & \quad \text{Time} \\
v & \quad \text{Speed of sound} \\
v_g & \quad \text{Group velocity} \\
w & \quad \text{Deflection / Gaussian beam radius} \\
\hat{x}, \hat{y} & \quad \text{Cartesian unit vectors} \\
x, y & \quad \text{Transverse Cartesian coordinates} \\
z & \quad \text{Axial position}
\end{align*}
List of Publications

Journal papers


Conference on Optical Fibre Sensors (OFS-24), (Curitiba, Brazil, September 28, 2015).


Abstract

Stimulated Brillouin Scattering (SBS) is a non-linear interaction between two optical waves, commonly referred to as pump and signal, and a mediating acoustic wave. The effect is named after the French physicist Léon Nicolas Brillouin, and was first observed experimentally by Chiao et al. in 1964. In the early 90’s, Horiguchi and coworkers had demonstrated that the SBS interaction can be employed in remote, distributed sensing over standard optical fibers. The technique was based on the dependency between the resonance frequency of the SBS interaction and physical attributes of the fiber, namely temperature and tensile strain. By limiting the pump wave to a narrow pulse, local information regarding the strength of the SBS interaction can be extracted from the temporal response of signal gain. This arrangement is known as Brillouin time domain analysis (B-OTDA). Typical B-OTDA setups reach measurement ranges of tens of km, but are fundamentally restricted to meter-scale resolution.

Since the 1990’s, numerous studies in both academia and industry have been dedicated to enhancing the capabilities of SBS based fiber sensors, in terms of resolution, measurement range, accuracy and speed. One effective configuration is that of Brillouin optical correlation domain analysis (B-OCDA), first developed by Hotate and collaborators in 2000. The method relies on the relation between the amplitude of the acoustic wave and the cross-correlation between complex envelopes of the optical waves. In the original proposition, constant-magnitude pump and signal waves are jointly frequency-modulated by a common sine wave. The two counter-propagating waves remain correlated in discrete and narrow locations, known as correlation peaks. The SBS interaction is largely restricted to these peaks only, which may be as narrow as few mm. The unambiguous analysis requires, however, that only a single peak is present along the fiber. Due to the inherent periodicity of the sine wave frequency modulation, the measurement range was initially restricted to few meters only. SBS over the entire fiber was mapped by sequentially moving the correlation peak over all points, one at a time.
In the work presented here, we propose a new approach to SBS sensing, which supports few cm resolution over several km of fiber. Our approach is based on the B-OCDA principle, but expands the modulation format to increase the separation between adjacent correlation peaks. To that end, we modulate the pump and signal waves by a common pseudo random phase code. The resolution is shown to depend only on the duration of individual symbols, whereas the separation between correlation peaks depends on the length of the sequence. Since very long random codes are readily available, the separation between correlation peaks can be arbitrarily long, thus lifting the fundamental range limitation of B-OCDA.

Analysis, numerical simulations and laboratory experiments show that the correlation peak may be placed at arbitrary locations along a fiber under test, allowing for random-access addressing of points of interest along the fiber. A complete scan of a 40 m long fiber with 1 cm resolution is demonstrated, and a 5 cm long heated section towards the end of the fiber is properly resolved. The measurement uncertainty corresponds to variations of ±0.5 °C, or 10 micro-strain. The 4,000 individual points addressed in this experiment were, at the time, the largest data set acquired by a B-OCDA setup.

The confinement of the acoustic field to discrete peaks is also used to form dynamic gratings in a polarization maintaining (PM) fiber. The gratings are inscribed by two phase-coded waves, polarized along one principal axis of the PM fiber. With careful frequency matching, probe signals polarized along the orthogonal principal axis are reflected by the gratings. The gratings are characterized through the analysis of reflected waveforms, in terms of their reflectivity, spatial width, spectral bandwidth and the extent of residual off-peak interactions. We find that in order to maintain sufficient levels of signal integrity, the noise caused by off-peak interactions must be reduced. Use of special binary phase codes with favorable correlation properties, known as ‘perfect Golomb codes’, was shown to reduce the overall off-peak reflectivity by a factor of 3-4. Dynamic gratings were successfully employed in the variable delay of 1 Gb/s data by as much as 10 ns, and with a sufficiently high signal-to-noise ratio to retain an open eye diagram.
The extension of measurement range in phase-coded B-OCDA was challenging due to two main reasons. First, the noise accumulated from off-peak interactions scales with fiber length. Second, each point must be addressed separately, making data acquisition unrealistically long. A new scheme for distributed Brillouin sensing that combines between time-domain and correlation-domain analysis principles addresses both issues. The Brillouin pump and signal waves are repeatedly co-modulated by a relatively short, high-rate phase sequence, which introduces Brillouin interactions in a large number of discrete correlation peaks. In addition, the pump wave is also modulated by a single amplitude pulse, which introduces a temporal separation between the stimulation of SBS at different peaks. The Brillouin amplification of the signal wave at individual peak locations is resolved in the time domain.

The technique provides the high spatial resolution and long range of unambiguous measurement offered by B-OCDA, together with reduced acquisition time through the sequential interrogation of a large number of resolution points by a single pulse. Using the proposed scheme, the Brillouin gain spectrum is mapped along a 1600 m-long fiber under test with a spatial resolution of 2 cm. All 80,000 resolution points are addressed with only 127 scans per choice of frequency offset between pump and signal.

Further increasing in measurement range is achieved with a method based on dual-layer, hierarchal encoding of both amplitude and phase. The pump and signal waves are again co-modulated by a relatively short, high-rate binary phase sequence which introduces Brillouin interactions in a large number of discrete and localized correlation peaks. In addition, the pump wave is also amplitude-modulated by a slower, carefully synthesized, long on-off-keying sequence. Brillouin interactions at the correlation peaks imprint weak replicas of the pump amplitude sequence on the intensity of the output signal wave. The Brillouin amplifications at individual correlation peaks are resolved by radar-like, matched-filter processing of the output signal.
Compression of the extended pulse sequence enhances the measurement signal-to-noise ratio, which is equivalent to that obtained by averaging over a large number of repeating single-pulse acquisitions. The acquisition times are potentially much reduced, and the number of resolution points that may be practically interrogated increases accordingly. In addition, the peak power level of the pump pulses may be lowered. Hence, the onset of phase pattern distortion due to self-phase modulation is deferred, and the measurement range can be increased. Using the proposed method, the acquisition of Brillouin gain spectra over a 8.8 km-long fiber with a spatial resolution of 2 cm is demonstrated experimentally. The entire set of 440,000 resolution points is interrogated using only 211 position scans per choice of frequency offset between pump and signal. A 7 cm-long hot-spot, located towards the output end of the pump wave, is properly recognized in the measurements. The number of addressed points is the largest of any Brillouin sensor at the time of writing.

Finally, we introduce an optomechanical fiber sensor, which addresses liquids outside the cladding of standard, 8/125 µm single-mode fibers with no structural modifications. Measurements are based on forward stimulated Brillouin scattering by radial, guided acoustic modes of the fiber structure. The acoustic modes are stimulated by an optical pump pulse and probed by an optical signal wave, both confined to the core. The acoustic vibrations induce a non-reciprocal phase delay to the signal wave, which is monitored in a Sagnac interferometer loop configuration. The measured resonance frequencies and the excitation strengths of individual modes agree with the predictions of a corresponding, quantitative analysis.

The acoustic reflectivity at the outer cladding boundary and the acoustic impedance of the surrounding medium are extracted from cavity lifetime measurements of multiple modes. The acoustic impedances of deionized water and ethanol are measured with better than 1% accuracy. The measurements successfully distinguish between aqueous solutions with 0, 4%, 8% and 12% concentrations of dissolved salt.
1 Introduction

1.1 Distributed sensing

Sensors are an integral part of any system, from large scale structures to microchips. Generally speaking, a sensor will convert a physical quantity of interest to a detectable signal. That signal, once detected, can convey information of numerous types, such as room temperature, blood pressure, light intensity and many more.

In many cases, multiple sensors are required to properly monitor a system, either due to its size or due to the complexity of the measured environment. For example complex materials in the aerospace industry are regularly exposed to extreme conditions of erosion and fatigue. Their integrity is, of course, crucial to the functionality and safety of the aircraft and therefore minor irregularities must be detected and diagnosed before deteriorating into a catastrophic failure. Because defects can potentially occur in any part, a network of sensors is required to cover any given point in the entire volume.

1.2 Distributed fiber sensing

From an engineering standpoint, a network of sensors must include the following:

- Large scale integration of multiple 'sensing nerves' into the structure under-test.
- A communication link to each sensor.
- A power supply for each sensor (for active sensors).

A large network of active electronic sensing devices, therefore, may be cumbersome to install and expensive to manufacture and maintain. In view of that, the motivation for fiber based sensors is clear: optical fibers can relay data over large distances without repeaters, they are relatively inexpensive and can be readily embedded into various environments. Moreover, they provide a unique added-value
by which every point along the link can become a potential sensor, while the entire fiber acts as the channel of communication for the network as a whole.

The concept of distributed fiber-based sensors has been explored in academic research and in industrial development over the past three decades. The first sensors were based on a strong optical pulse that was reflected back to a detector through Rayleigh backscatter [1]. Such systems are named optical time domain reflectometers (OTDRs). Commercially available OTDRs were first used to detect lossy links and fiber defects. In later years, environmental modifications to the Rayleigh backscatter were used to sense temperature and strain in a truly distributed manner. Modern systems can detect both strain and temperature with sub-mm resolution along hundreds of meters [2]. Other systems can cover up to 100 km with meter-scale resolution [3]. Acquisition time was reduced to the order of ms, enabling real-time distributed acoustic sensing of up to hundreds of Hz [4]. On-going research is dedicated to further reducing acquisition time, while improving measurement range [5, 6], resolution [5-7] and sensitivity [8].

Rayleigh backscatter is only one example of a physical process that affects the scattering of light waves in an optical fiber. In the 30 years of research, several other mechanisms were also employed in the field of optical fiber sensing, e.g. stimulated Brillouin scattering (SBS), Raman scattering and fiber Bragg gratings (FBGs). Each of the above represents a unique field of research, having its own advantages and limitations. However different, a distinct similarity exists between all these methods: they are based on the scattering of light. While light reflected by Rayleigh backscatter and FBGs is at the same frequency as that of the incident beam (‘elastic scattering’), the processes of SBS and Raman scattering are associated with a frequency shift (‘inelastic scattering’). Scattering properties such as reflectivity, wavelength of maximum scatter and frequency shift (when relevant) depend on the local physical attributes, mainly temperature and strain, which can be resolved via detection of the scattered light.

In this work, I study various aspects of SBS-based sensing, and it is thus discussed in depth. In the following section I will briefly survey the other three sensing
methods mentioned above, namely those based on Rayleigh, Raman and FBG-induced scattering.

1.2.1 Rayleigh scattering-based fiber sensors

Rayleigh backscatter is an elastic process, in which density fluctuations on sub-wavelength scale cause an incident light wave to scatter in all directions. Some of the light is directly backscattered in angles that fall within the numerical aperture of the fiber, recaptured and allowed to propagate in the opposite direction [9]. This interaction between light and matter is ‘linear’, in the sense that the scattered light intensity is directly proportional to the intensity of the incident light.

Figure 1: Illustration of Rayleigh scattering [9].

Figure 1 illustrates the process of Rayleigh scattering. A local variation in density leads to a disturbance in the dielectric constant of the fiber core ($\Delta \varepsilon$), and therefore to a local refractive index change. The temporal pattern of the scattered light is determined by the spatial distribution of refractive index. That spatial pattern varies when either temperature or strain in the fiber are perturbed. The unique spatial pattern of index variations may be unpredictable, but can be compared against a reference measurement taken with the same fiber. Any changes in the pattern can then be interpreted as temperature or strain perturbations.

A mathematical description of Rayleigh scattering is given in [9], and will be repeated here briefly. Consider a continuous wave (CW) with an angular frequency $\omega$ propagating in the positive $z$ direction. The complex envelope of the electric field backscattered from fiber of length $z$ is approximately given by the equation [9]:

$$A(z, \beta) \approx \frac{\beta E_0}{2i} \int_0^z \frac{\Delta \varepsilon(z')}{\varepsilon} e^{2i\beta z'} dz'.$$  \hspace{1cm} (1.1)
In (1.1), $\beta$ is the propagation constant of the field ($\beta = \omega \sqrt{\mu_0 \varepsilon}$), $E_0$ is the amplitude of the incident field, $\varepsilon$ is the nominal dielectric constant of the core and $\Delta \varepsilon$ represents the dielectric constant variations that accompany the density fluctuations. As can be seen from eq. (1.1), the reflected field is proportional to the spatial Fourier transform of the density variations. One type of sensor that takes advantage of this is an optical frequency domain reflectometer (OFDR) [10]. In OFDR setups (see Figure 2), a CW wave is split into two branches, one branch launches the wave into a fiber under test and the second is kept as reference. The interference between the reflected wave and the local reference gives both the magnitude and phase of the Fourier transform component at a single frequency. The spatial density distribution is resolved by scanning the CW frequency and performing an inverse Fourier transform.

![Figure 2: Schematic description of an OFDR setup [10].](image)

The resolution of commercial OFDR systems can be on the mm scale [10], limited by the span of frequency sweep that is supported by the laser source. For example, a spatial resolution of 1 mm requires the scanning frequency span of 100 GHz (about 0.8 nm). High-end OFDR systems exhibit the best spatial resolution of all fiber based sensor types, resolving details as small as 10 μm. In recent studies, dynamic OFDRs successfully detected vibrations up to 2 kHz rates along 50 km of fiber [11]. There are however a few limitations to their employment, two of which will be mentioned here. First, a single read of the density fluctuations gives no information
regarding the temperature or strain along the fiber. In order to resolve them, the local density must be compared against a reference measurement, taken with the same fiber and at exactly the same external conditions. Second, measurement of both phase and magnitude of the reflected wave requires coherent detection and it is therefore limited by the coherence length of the laser source. High resolution OFDRs reach about 100 m of sensing length [10]. Nevertheless, OFDRs are extensively researched in academia and are also widely acknowledged by industry.

A different type of a Rayleigh scattering-based sensor is the OTDR. It detects the Rayleigh back-reflection of a short pulse, rather than a CW wave. The OTDR trace is equivalent to measuring the impulse response of the fiber, rather than its frequency response. Naturally, the resolution of such setups would depend on the length of the pulse – the shorter the better. The relatively low reflectivity of Rayleigh backscatter (approximately -100 dB per millimeter) sets a lower bound on the length of detectable pulses. Commercial OTDR systems with relatively low resolution of 1 meter are successfully employed in telecommunication applications for link maintenance and fault detection. Higher resolution setups use more elaborate detection schemes, by which a resolution on the order of 1 cm was demonstrated [12].

1.2.2 Raman scattering-based sensors

In contrast to Rayleigh backscatter, Raman scattering is a non-parametric, nonlinear process. It describes the excitation of molecules to high order vibrational states by incident photons and the subsequent emission of photons at different wavelengths [13]. Figure 3 illustrates the possible decay paths of an excited molecule. In Rayleigh backscatter, a molecule in the ground state (‘0’) decays back to its original state and the absorbed photon is at the same wavelength as the one emitted. However, in some cases the molecule may decay to a higher state (state ‘1’). In that case the emitted photon will be lower in energy and longer in wavelength and is referred to as a ‘Stokes’ photon. In a different, less likely case, the molecule would start the interaction at state ‘1’ and decay to the ground state, emitting a photon of higher energy called an ‘anti-Stokes’ photon. The wavelength shift between absorbed and emitted photons in Raman scattering corresponds to the energy difference
between the molecular vibrational states. It corresponds to a wavelength difference on the order of 80 nm in silica fibers at telecommunication wavelengths.

The ratio between the intensities of Stokes and anti-Stokes spontaneous scattering is:

$$\frac{I_{AS}}{I_S} = \left(\frac{\lambda_S}{\lambda_{AS}}\right)^4 \exp\left(-\frac{\hbar \omega_M}{k_B T}\right)$$

(1.2)

In (1.2), $I_{AS}$ and $I_S$ are the anti-Stokes and Stokes waves intensities respectively, $\lambda_{AS}$ and $\lambda_S$ are the ant-Stokes and Stokes wavelengths respectively, $\hbar$ is Planck’s constant divided by $2\pi$, $k_B$ is Boltzmann’s constant, $T$ is the temperature and $\omega_M$ is the frequency of the quantum oscillator associated with the molecule. The key to Raman scattering-based sensing lies in the fact that the above ratio explicitly depends on the fiber temperature $T$. Hence, the ratio of spontaneous Raman Stokes and anti-Stokes scattering can be measured to gain information related to temperature of remote locations in the fiber. Distributed temperature sensing of a 135 m-long fiber with 24 cm spatial resolution was recently reported [15]. Range extension methods reached up to 40 km [16] with temperature sensitivity of 5 degrees and 17 m resolution. The main limiting factor of these sensors is the low reflectivity of
spontaneous Raman scattering, requiring the use of relatively long interrogation pulses which degrade resolution. In contrast to other fiber sensing techniques, Raman scattering is completely unaffected by strain. That is both a potential drawback and an advantage. In cases where strain is an important quantity to measure, it is of course impossible to use Raman sensing. On the other hand, in Rayleigh, FBG and Brillouin sensing both strain and temperature changes manifest in a similar manner, resulting in ambiguous measurements, a problem that does not exists in Raman based sensors.

1.2.3 Fiber Bragg gratings-based sensors

An FBG is a periodic refractive index perturbation inscribed in the fiber core by ultraviolet (UV) illumination. The condition for maximum reflection from the grating, called the ‘Bragg condition’, is [17]:

\[ \lambda_b = 2n_{\text{eff}} \Lambda \]  

(1.3)

In (1.3) \( n_{\text{eff}} \) is the effective refractive index of the fiber mode, \( \Lambda \) is the spatial period of the grating and \( \lambda_b \) is the optical wavelength of maximum reflectivity, also known as the ‘Bragg wavelength’. As can be expected, both the period \( \Lambda \) initially inscribed and the effective index can vary slightly when the temperature and strain of the fiber are modified [18]. Those variations can be optically interrogated through the Bragg condition. At 1550 nm, small strain variations cause a shift in the Bragg wavelength at a ratio of \( 1.2 \text{ pm/\mu\varepsilon} \). Temperature changes also shift the Bragg wavelength through the thermo-optic effect and linear thermal expansion, at a rate of \( 10 \text{ pm/°C} \) at 1550 nm [18].

It is important to note that FBG-based sensors, unlike other fiber sensors, are not truly distributed. A single FBG can only monitor the environmental conditions in close proximity to its location. However, multiple gratings can be inscribed into a single fiber, each with its own unique period (see Figure 4). This allows for the integration of many point sensors into a ‘quasi-distributed network’. While this approach allows FBGs to be deployed in scenarios where distributed sensing is necessary, inscription of the gratings often involves the removal of the polymer jacket,
leaving the fiber mechanically vulnerable. Despite that, FBGs are known as reliable and simple sensors, commonly employed in commercial applications.

![Figure 4: Illustration of multiple FBGs interrogated by a broadband signal (from [4]).](image)

1.3 Stimulated Brillouin Scattering

In the previous section, I listed several mechanisms that underlie distributed fiber sensing. In my research, I have dealt exclusively with sensors based on SBS. In this section I elaborate on the physical mechanisms of SBS and derive the governing partial differential equations (PDEs) of SBS interactions. This section is based primarily on *Nonlinear Optics* by R. W. Boyd, 3rd Edition, Academic press, 2008 [13].

1.3.1 Brillouin Scattering – Introduction

Brillouin Scattering, named after the French physicist Léon Brillouin, is an interaction between two counter-propagating optical waves and an acoustic wave. In spontaneous Brillouin scattering, an optical wave is ‘scattered’ by an acoustic wave. Since the acoustic wave travels at the speed of sound, the frequency of the scattered optical wave is Doppler shifted by a quantity named the ‘Brillouin frequency shift’: \( \nu_b \sim 10–11 \text{ GHz} \) (in standard silica fibers and at 1550 nm). Thermally excited acoustic waves naturally exist in any material at non-zero temperature. On the other hand, the process is said to be ‘stimulated’ when the acoustic waves themselves are excited by the optical waves, through a mechanism called ‘electrostriction’. In most conditions, the scattering itself causes the electrostriction to intensify, resulting in a positive feedback loop (see Figure 5).
Three main mechanisms come into play in SBS:

1. Electrostriction
2. The acousto-optic effect
3. Scattering by a moving grating

Each of the above will be explained in detail in section 1.3.2. The concept of frequency matching and its implications on the Brillouin frequency shift will also be discussed.

From an optical perspective, SBS is described as a nonlinear interaction since the refractive index changes in reaction to the presence of optical waves, even if indirectly. Three coupled PDEs can be derived from the optical and acoustic wave equations. The evolution of the three waves can be predicted by solving these equations, either numerically or analytically. The equations and their solutions in few specific cases will be derived in section 1.3.3.

### 1.3.2 SBS – Physical Mechanisms

#### 1.3.2.1 Electrostriction and the acousto-optic effect

Consider an electric dipole $\mathbf{p} = q\mathbf{L}$, where two point charges, $\pm q$, are located at locations $\mathbf{r}_1, \mathbf{r}_2$, separated by a small distance $\mathbf{L}$. The electromagnetic (EM) force applied to the dipole, known as the Lorentz force, is given by:
\[ F = q \left( \mathbf{E}(r_1) - \mathbf{E}(r_2) + \frac{d(r_1 - r_2)}{dt} \times \mathbf{B} \right) \tag{1.4} \]

In (1.4), \( \mathbf{E} \) and \( \mathbf{B} \) are the electric and magnetic fields, respectively. The definition for the directional derivative along vector \( \hat{\mathbf{L}} \) is:

\[ (\hat{\mathbf{L}} \cdot \nabla) \mathbf{E}(r_2) = \lim_{h \to 0} \frac{\mathbf{E}(r_2 + h\hat{\mathbf{L}}) - \mathbf{E}(r_2)}{h} \approx \frac{\mathbf{E}(r_1) - \mathbf{E}(r_2)}{L} \tag{1.5} \]

Here \( \hat{\mathbf{L}} \) is the unit vector along \( \mathbf{L} \), and \( L \) denotes the length of \( \mathbf{L} \). The approximation in (1.5) holds for \( L \to 0 \), which is true for a point dipole. Using this notation, eq. (1.4) can be rewritten as follows:

\[ F = q \left( L (\hat{\mathbf{L}} \cdot \nabla) \mathbf{E} + \frac{\partial}{\partial t} \mathbf{d} \times \mathbf{B} \right) = \left( \mathbf{p} \cdot \nabla \right) \mathbf{E} + \frac{\partial}{\partial t} \mathbf{p} \times \mathbf{B} \tag{1.6} \]

If the dipole in question is a polarized molecule, the dipole moment can be assumed to be linearly proportional to the electric field: \( \mathbf{p} = \varepsilon_0 \alpha \mathbf{E} \), where \( \alpha \) is the polarizability of the molecule. Using a vector analysis identity and one of Maxwell’s equations, we have:

\[ F = \varepsilon_0 \alpha \left[ \frac{1}{2} \nabla E^2 + \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B}) \right] \tag{1.7} \]

The second term in (1.7) is a time derivative of the ‘Pointing vector’ of the EM radiation. In the optical spectrum, the frequency of EM radiation is in the order of \( \nu \sim 10^{14} \) Hz, hence the force applied by this term will average to zero over any time constant relevant to acoustic waves. However, the first term in (1.7) is relative to the gradient of the square magnitude of the electric field and for that reason, even for rapidly oscillating waves, its time average may be non-zero. A dipole will always be pulled into the region of increased field strength, regardless of the direction of the field. The above derivation is made for a single molecule, but can be easily expanded to a large ensemble of molecular dipoles, namely – a dielectric material.
The force applied by the electric field will cause a change in the local density of the dielectric material $\Delta \rho$. The relative permittivity $\varepsilon_r$ depends on the density of the medium. For small changes we can assume a linear connection between the density fluctuations $\Delta \rho$ and the permittivity fluctuations $\Delta \varepsilon$:

$$\Delta \varepsilon = \left( \frac{\partial \varepsilon_r}{\partial \rho} \right) \Delta \rho$$

Consequently, the electric field energy density changes by:

$$\Delta u = \frac{1}{2} \varepsilon_0 E^2 \Delta \varepsilon = \frac{1}{2} \varepsilon_0 E^2 \left( \frac{\partial \varepsilon_r}{\partial \rho} \right) \Delta \rho$$

On the other hand, the added energy must come from the work performed during the compression of the material. This work can be formulated in terms of an electrostrictive pressure $p_e^s$:

$$\Delta w = -p_e^s \frac{\Delta \rho}{\rho}$$

Comparing eq. (1.9) and (1.10), we find that the electrostrictive pressure is:

$$p_e^s = -\frac{1}{2} \varepsilon_0 \gamma_e E^2$$

In (1.11) we have introduced the electrostrictive constant $\gamma_e$, defined as:

$$\gamma_e = \rho \frac{\partial \varepsilon_r}{\partial \rho}$$

The change in density is given by:

$$\Delta \rho = \frac{1}{2} \varepsilon_0 \rho C \gamma_e E^2$$

where $C = \rho ^{-1} \left( \partial \rho / \partial \rho \right)$ is the compressibility of the medium. In this derivation it was implicitly assumed that the fields are static. When dealing with time varying fields in the optical spectrum, a time average must be taken:
\[ \Delta \rho = \frac{1}{2} \varepsilon_0 \rho C \gamma_e \left\langle E^2 \right\rangle \]  

(1.14)

In (1.14) the brackets indicate averaging over a large number of optical periods.

From the above derivation we learn that electric fields in a dielectric medium can induce density fluctuations. But, as mentioned, density variations must also cause changes to the permittivity, and through that to the refractive index \( n = \sqrt{\varepsilon \mu} \). Optical waves are affected by those refractive index perturbations and may be scattered by them coherently, if certain conditions are met (see next section). That process is often referred to as the acousto-optic effect.

### 1.3.2.2 Acoustic frequency matching

In the previous section the basic physical concepts behind light-sound interactions were discussed. In this section, it is shown that the interaction will reinforce itself through positive feedback if the frequency detuning between the two optical signals matches the Brillouin frequency. These conditions are called frequency-matching conditions.

Consider two optical waves, \( \vec{E}_1 \) and \( \vec{E}_2 \) with frequencies \( \omega_1 \) and \( \omega_2 \), respectively (\( \omega_1 > \omega_2 \)), where \( \vec{E}_1 \) propagates in the positive \( z \) direction and \( \vec{E}_2 \) in the negative \( z \) direction:

\[ \vec{E}_1(t, z) = A_1 e^{i(k_1z - \omega_1t)} + \text{c.c.} \]  

(1.15)

\[ \vec{E}_2(t, z) = A_2 e^{i(k_2z - \omega_2t)} + \text{c.c.} \]  

(1.16)

Here \( A_1 \) and \( A_2 \) are complex envelopes, taken to be constants for the purpose of this discussion. \( k_i = n \omega_i / c \) are the wave numbers of the two waves, \( i = 1,2 \), \( c \) is the speed of light in vacuum, \( n \) is the index of refraction and \( t \) denotes time. The two optical waves are taken to be linearly polarized along the same axis, so that the overall electric field \( \vec{E} \) will be the algebraic sum of the two. Substituting \( \vec{E} \) in (1.14), we obtain the induced material density variations:
\[ \Delta \rho \sim \frac{1}{2} \left( (A_1 \cos(k_1 z - \omega_1 t) + A_2 \cos(k_2 z + \omega_2 t))^2 \right) \]
\[ = A_1^2 + A_2^2 + A_1 A_2 \left[ \cos((k_1 + k_2)z - (\omega_1 - \omega_2) t) \right] \]

(1.17)

As can be directly observed form the result of (1.17), the density variations form a traveling acoustic wave, with frequency \( \Omega \) and wave number \( q \):

\[ \Omega = \omega_1 - \omega_2 \]
\[ q = k_1 + k_2 \]

(1.18)

Eq. (1.18) can be interoperated from a quantum mechanical perspective: a photon with energy \( \hbar \omega_1 \) is annihilated to create a photon with energy \( \hbar \omega_2 \) and a phonon with energy \( \hbar \Omega \). The first equation in (1.18) therefore represents the conservation of energy in the process. Photons and phonons also carry momenta \( p = \hbar k \), so the second equation represents the conservation of momentum.

From (1.18) we see that the optically-stimulated acoustic perturbations are forced to travel at a phase velocity of \( \bar{v} = \Omega / q \). However, density variations must obey the acoustic wave equation (see eq. (1.30) ahead) and therefore travel at the speed of sound \( v \). Intuitively speaking, if \( \bar{v} \neq v \), contradicting constraints are placed on the stimulated acoustic wave. This argument will be mathematically substantiated in the next section. Subject to the requirement of \( v = \bar{v} \), equations (1.18) can be solved simultaneously for \( \Omega \) in terms of \( \omega_1 \):

\[ \Omega = \Omega_g = \frac{2\nu}{\omega_1} \frac{c/n}{1 + \frac{\nu}{c/n}} \]

(1.19)

\( \Omega_g \) is called the Brillouin frequency shift. Given that the speed of sound \( v \) is significantly smaller than the speed of light, Eq. (1.19) can be approximated as:

\[ \Omega_g = \frac{2\nu}{c/n} \omega_1 \]

(1.20)

Using the same approximation, the acoustic wave vector is given by:
\[ \mathbf{q}_b = 2\mathbf{k}_1 \quad (1.21) \]

To continue the intuitive observation, an acoustic wave can only be excited effectively if the optical waves are offset in frequency by \( \Omega_b \). Typical values of \( \Omega_b \) for fused silica at 1550nm wavelength are \( \sim 2\pi \times 11\text{GHz} \).

### 1.3.2.3 Acousto-optic diffraction and Doppler shift

For the next discussion, consider an acoustic wave with frequency \( \Omega \) and wave vector \( \mathbf{q} \) propagating along the positive \( z \) direction:

\[ \tilde{\rho}(z,t) = \rho_0 + \left[ \rho(z,t)e^{i(qz-\Omega t)} + \text{c.c.} \right], \quad (1.22) \]

where \( \rho_0 \) denotes the mean density of the medium. The density variations will be accompanied by refractive index variations:

\[ \tilde{n}(z,t) = n_0 + \left[ \Delta n(z,t)e^{i(qz-\Omega t)} + \text{c.c.} \right] \quad (1.23) \]

where \( n_0 \) is the intrinsic refractive index of the medium and \( \Delta n \) is the amplitude of index variations, given by:

\[ \Delta n = \frac{\gamma}{2n_0 \rho_0} \Delta \rho \quad (1.24) \]

As mentioned in section 1.2.3, optical waves can be reflected by periodic refractive index changes (namely, a Bragg grating), given that the optical wavelength matches the diffraction phase-matching condition (repeated here for convenience):

\[ \lambda_b = 2n_0 \Lambda \quad (1.25) \]

Note that the condition in Eq. (1.25) matches exactly the one given in (1.21). In other words, an acoustic wave induced by phase-matched electrostriction will naturally act as a Bragg reflector for the \( \omega_1 \) wave. Once the acoustic wave builds up, the higher frequency wave (which will be named from this point onwards the ‘pump’ wave) will be reflected back by the acoustic wave.
An acousto-optic reflector, unlike a standard FBG, moves at the speed of sound. The frequency of waves scattered by a moving reflector is Doppler shifted by an amount:

\[ \Delta \omega = \frac{2n v}{c} \omega_2 \]  

(1.26)

Comparing this result with Eq. (1.20), we see that:

\[ \Delta \omega = \Omega_B \]  

(1.27)

Since the direction of the induced acoustic wave is the same as that of the pump, the Doppler shift will be a ‘red’ shift, so that the frequency of the reflected wave will be lower than the frequency of the pump by \( \Omega_B \) and exactly match \( \omega_2 \), the frequency of the wave we shall call ‘probe’.

To summarize, effective acoustic wave build-up through electrostriction requires matching between the phase velocity of the optically-induced ‘driving force’ and the speed of sound. This condition imposes a specific detuning between the pump and probe frequencies, by the Brillouin frequency shift. Once those conditions are met, the acoustic wave will naturally act as moving Bragg reflector for the pump wave, and the reflected signal will be Doppler shifted to the frequency of the probe. The probe will then gain power at the expense of the pump. The entire process is called **stimulated Brillouin scattering** (SBS).

### 1.3.3 Coupled Wave Equations

We again consider two counter-propagating optical waves, detuned in frequency by \( \Omega \):

\[ \tilde{E}_1(z,t) = A_1(z,t)e^{i(k_1z-\omega_1t)} + \text{c.c.} \]  

(1.28)

\[ \tilde{E}_2(z,t) = A_2(z,t)e^{i(k_2z-\omega_2t)} + \text{c.c.} \]  

(1.29)

Through electrostriction, an acoustic field will be induced in the medium. The governing wave equation for the propagation of the density \( \tilde{\rho}(z,t) \) is:
\[ \frac{\partial^2 \tilde{\rho}}{\partial t^2} - \Gamma' \nabla^2 \frac{\partial \tilde{\rho}}{\partial t} - \nu^2 \nabla^2 \tilde{\rho} = \nabla \cdot \mathbf{f}, \]  \hspace{1cm} (1.30)

where \( \Gamma' \) is the acoustic damping parameter. The non-homogeneous term on the right-hand side is the divergence of the electrostrictive force, given by:

\[ \mathbf{f} = \nabla \rho_{st}, \quad \rho_{st} = -\frac{1}{2} \varepsilon_0 \gamma_e \langle \tilde{E}^2 \rangle \]  \hspace{1cm} (1.31)

For the fields given above, the source term is of the form:

\[ \nabla \cdot \mathbf{f} = \varepsilon_0 \gamma_e q^2 \left[ A_2^* A_2 e^{i(qz - \omega t)} + \text{c.c.} \right] \]  \hspace{1cm} (1.32)

Since the wave equation represents a linear system driven by a harmonic function, we can infer that the solution (or ‘output’ of the system) will be a harmonic function:

\[ \tilde{\rho}(z,t) = \rho_0 + \left[ \rho(z,t) e^{i(qz - \omega t)} + \text{c.c.} \right]. \]  \hspace{1cm} (1.33)

Introducing (1.33) into (1.30), we get:

\[ \frac{\partial \rho}{\partial t} + \Gamma_A \rho + \nu \frac{\partial \rho}{\partial z} = jg A^*_2, \]  \hspace{1cm} (1.34)

where it was assumed that the amplitude of the acoustic field varies slowly, that is:

\[ \left| \frac{\partial^2 \rho}{\partial t^2} \right| \gg \left| \frac{\partial \rho}{\partial t} \right|, \quad \left| \frac{\partial^2 \rho}{\partial z^2} \right| \gg \left| \frac{\partial \rho}{\partial z} \right|. \]  \hspace{1cm} (1.35)

In (1.34), \( \Gamma_A \) is the frequency dependent complex damping factor:

\[ \Gamma_A = j \frac{\Omega_A^2 - \Omega^2 - j \Omega \tau}{2 \Omega}, \]  \hspace{1cm} (1.36)

where the Brillouin linewidth, which is also the reciprocal of the acoustic life time \( \tau \), was introduced.
\[ \Gamma_s = q^2 \Gamma' = \frac{1}{\tau} \] (1.37)

On the right hand-side of (1.35), \( g_1 \) is given by:

\[ g_1 = \frac{\varepsilon_0 \gamma_q q}{2v} \] (1.38)

Eq. (1.34) can be further simplified by neglecting the spatial derivative. The justification for that comes from the relatively short life time of phonons. At a typical Brillouin frequency of \( \Omega_b = 2\pi \times 11\text{GHz} \), the phonon life time is about \( \tau \sim 5\text{ ns} \) and the propagation length is \( \sim 25\text{ \mu m} \) (at \( v \sim 5000\text{ m/s} \)). If the complex envelope can be assumed to be constant within that segment than the spatial derivative can be safely neglected.

The optical fields evolve according to their respective nonlinear wave equations:

\[
\frac{\partial^2 \vec{E}_i}{\partial z^2} - \frac{1}{(c / n)^2} \frac{\partial^2 \vec{E}_i}{\partial t^2} = \frac{1}{\varepsilon_0 c^2} \frac{\partial^2 \vec{P}_i}{\partial t^2}, \quad i = 1, 2
\] (1.39)

where \( \vec{P}_i \) are the nonlinear polarization terms that match the frequency and wave vector of each field component. The overall nonlinear polarization is given by:

\[ \vec{P} = \varepsilon_0 \Delta \gamma \vec{E} \] (1.40)

Using the definition of the electrostriction coefficient, and noting that \( \Delta \chi = \Delta \varepsilon \), we find that:

\[ \vec{P} = \varepsilon_0 \rho_0^{-1} \gamma_e \vec{P} \vec{E} \] (1.41)

Only nonlinear polarization terms that oscillate at the correct frequency and wavenumber can act as phase-matched source terms. Those terms are:

\[
\vec{P}_1 = p_1 e^{i(k_z x - \omega t)} + c.c.
\]
\[
\vec{P}_2 = p_2 e^{i(-k_z x - \omega t)} + c.c.
\] (1.42)
where the complex magnitudes of the nonlinear polarization terms are:

\[ p_1 = \varepsilon_0 \gamma \rho_0^{-1} \rho A_2 \]
\[ p_2 = \varepsilon_2 \gamma \rho_0^{-1} \rho^* A_1 \]  

(1.43)

Introducing Eq. (1.42) into (1.39) and taking the slowly varying amplitude approximation yet again, we obtain:

\[ \frac{\partial A_1}{\partial z} + \frac{1}{c/n} \frac{\partial A_1}{\partial t} = g_2 \rho A_2, \]

(1.44)

and

\[ -\frac{\partial A_2}{\partial z} + \frac{1}{c/n} \frac{\partial A_2}{\partial t} = g_2 \rho^* A_1, \]

(1.45)

where:

\[ g_2 = \frac{\omega \gamma}{2nc^2 \rho_0}, \]

(1.46)

and \( \omega = \omega_1 \approx \omega_2. \)

Eqs. (1.44) and (1.45), along with Eq (1.34), fully describe the temporal and spatial evolution of the complex envelopes of the three coupled waves in SBS: the pump \( A_1 \), the probe \( A_2 \) and the acoustic field \( \rho \). Their general solution for arbitrary boundary condition can only be evaluated numerically. However, a steady-state solution for undepleted pump and a semi-analytic general solution can be obtained.

1.3.3.1 Steady state solution to the coupled wave equations

Consider a solution in which the pump and probe are continuous waves (CWs) at their respective input ports. This is a special case, with seemingly little relevance to sensing schemes, but it does give an important intuitive perspective. In this case, all waves will reach a steady state after a sufficiently long time, (in accordance with the length of the fiber and the acoustic life time), and all temporal derivatives can be dropped from the coupled wave equations. Since the spatial derivative are also neglected in Eq. (1.34), a solution for \( \rho \) can be trivially obtained:
\[
\rho(z) = j \frac{g_z}{\Gamma_A} A_z A_{z*}^*
\]  

(1.47)

The nonlinear wave equations for the evolution of pump and probe may be reformulated in terms of electro-magnetic intensities, in W per unit area, rather than field magnitudes. Defining the intensities as:

\[
l_i = 2\varepsilon_0 c |A_i|^2,
\]  

(1.48)

we find that:

\[
\frac{dl_1}{dz} = -g l_1 l_2
\]  

(1.49)

and

\[
\frac{dl_2}{dz} = -g l_1 l_2.
\]  

(1.50)

In (1.49) and (1.50) the SBS gain factor was introduced. Its dependence on the frequency detuning between pump and signal is given to a good approximation by:

\[
g = g_0 \frac{(\Gamma_B / 2)^2}{(\Omega_B - \Omega)^2 + (\Gamma_B / 2)^2},
\]  

(1.51)

where the line-center gain factor is:

\[
g_0 = \frac{\gamma^2 \omega^2}{n \nu c^2 \rho \Gamma_B}.
\]  

(1.52)

Here we make another key assumption: it is assumed that the power gained by the probe (and lost by the pump) is negligible with respect to the incident power of the pump, throughout its propagation. That assumption is called the ‘undepleted pump approximation’. Under that condition \( l_1 \) is constant, and Eq. (1.50) can be readily solved:

\[
l_z(z) = l_z(L) e^{2g(l-z)},
\]  

(1.53)
where \( L \) is the length of the medium. In this special case, the probe experiences an exponential gain as it propagates through the medium. For fused silica fibers the line center gain is approximately \( g_0 = 7.5 \text{ m/GW} \). A more practical representation of Brillouin gain in fiber is in terms of the exponential growth of the probe optical power, rather than the intensity. Taking a core cross-section area of \( A_{\text{eff}} = \pi \times (6.5)^2 \mu \text{m}^2 \), we obtain \( g_0 = 0.1 \text{ m}^{-1} \text{W}^{-1} \).

Note that the exponential gain factor \( g \) is frequency dependent (see eq. (1.51)). It reaches a maximum value of \( g_0 \) when the frequency difference \( \Omega \) equals the Brillouin frequency shift. The full width at half maximum (FWHM) of the Lorenzian line-shape is \( \Gamma_0 \). Its value for standard silica fibers is on the order of \( 2\pi \cdot 30 \text{ MHz} \).

### 1.3.3.2 Semi-analytic solution using the method of characteristics

In this section I describe a method to reduce the three coupled partial differential equations (PDEs) to ordinary differential equations (ODEs). Although the ODEs are still coupled, a general solution in the form of an implicit integral can be derived. It plays an important role in the numerical analysis given in later chapters.

Eq. (1.34), after neglecting the spatial derivative term, can be solved using an integrating factor [19]. The solution is:

\[
\rho(z,t) = jg_1 \int_0^t e^{-T_s(t-t')} A_i(z,t')A_1^\ast(z,t') dt' + \rho(z,0) \tag{1.54}
\]

The similarity between this solution and a cross-correlation function between the two optical field envelopes at position \( z \) is the foundation upon which correlation-domain SBS sensing techniques are based. Its consequences are discussed at length in section 1.4.2.

Equations (1.44) and (1.45) may be reduced to ODEs using the method of characteristics [19]. We define a curve in time-space \( z(t) \) and formulate it by the parameter \( s \) such that the PDE reduces to an ODE with a single variable \( s \). Starting with (1.44):
\[
\frac{dA_1}{ds_1} = \frac{\partial A_1}{\partial z} + \frac{\partial A_1}{\partial t} = jg_2 \rho A_2
\] (1.55)

Comparing (1.55) with (1.44), we find that:

\[
\frac{\partial z}{\partial s_1} = 1 \rightarrow z = s_1 + z_{01} \\
\frac{\partial t}{\partial s_1} = \frac{1}{c/n} \rightarrow t = \frac{s_1}{c/n} + t_{01}
\] (1.56)

Using this curve, the solution can be found with direct integration:

\[
A_1(s_1) = \int_0^{s_1} \frac{i \omega \gamma e}{2nc \rho_0} \rho(s') A_2(s') ds' + A_1(s_1 = 0)
\] (1.57)

Using the same procedure on (1.45), we find:

\[
\frac{\partial z}{\partial s_2} = -1 \rightarrow z = -s_2 + z_{02} \\
\frac{\partial t}{\partial s_2} = \frac{1}{c/n} \rightarrow t = \frac{s_2}{c/n} + t_{02}
\] (1.58)

The solution will be of the form:

\[
A_1(s_2) = \int_0^{s_2} \frac{i \omega \gamma e}{2nc \rho_0} \rho^*(s') A_1(s') ds' + A_1(s_2 = 0)
\] (1.59)

### 1.3.4 Forward Brillouin scattering

In the discussion so far, the stimulating optical waves in SBS were taken to be counter propagating. In a paper published in 1985, Shelby and co-authors demonstrated that certain acoustic modes, guided by the cylindrical shape of the fiber cladding, can coherently scatter optical incident waves in the forward direction [20]. Further, as described by Dianov and co-authors [21], those specific modes can be excited by the electrostrictive pressure induced by the fundamental optical mode in a single-mode fiber.
For an intuitive insight as to the conditions required for phase-matched forward scattering by acoustic waves, it is helpful to re-write Eq. (1.18) in vector notation:

\[
\begin{bmatrix}
\omega_1 \\
k_1
\end{bmatrix} = \begin{bmatrix}
\Omega \\
q
\end{bmatrix} + \begin{bmatrix}
\omega_2 \\
k_2
\end{bmatrix}
\] (1.60)

Figure 6 illustrates (1.60) in the \( \omega \) vs. \( k \) plane for the case of backward scattering (left panel). As can be seen, for any ‘dark blue vector’, representing the pump wave, a pair of probe and acoustic wave vectors (in light blue and red, respectively) can be found such that the slope of the vectors match the light and sound velocities (represented in dashed lines) while meeting the condition of (1.60).

In contrast, when the optical waves co-propagate (Figure 6, right panel), the phase of the beating pattern will always travel at the speed of light, which is five orders of magnitude greater than the speed of sound. Interestingly (and somewhat unintuitively), some acoustic modes, guided by the cladding structure of the fiber, have arbitrarily small axial wave vectors near their cut-off frequency \( \Omega_c \), enabling arbitrarily high phase velocities. Such acoustic waves are phase-matched to any co-propagating pair of optical waves, with frequency a difference that matches \( \Omega_c \).

An Additional requirement for the effective stimulation of these acoustic modes of the entire fiber structure is that the electrostrictive force must have non-
vanishing spatial overlap with the transverse profile of the guided acoustic mode. Two categories of acoustic modes meet these conditions: purely radial modes and mixed torsional-radial modes. Since each mode has a unique cut-off frequency, scattering is not limited to a single ‘Brillouin frequency’, but rather to a set of frequencies, starting at approximately 30 MHz.

The conditions for forward Brillouin scattering and its potential application in fiber sensing are discussed in detail in Chapter 5.

1.4 SBS sensing

One of the key elements of SBS is the specific frequency offset between pump and probe required for an effective gain, known as the Brillouin frequency shift. Referring back to (1.20), we see that the Brillouin frequency depends on the speed of sound $v$ and speed of light $c/n$. Those properties, in turn, depend on the local physical state of the medium, mainly on temperature and strain, and therefore so does the Brillouin shift [22]:

$$\nu_B = \nu_{B,0} + C_T \Delta T + C_\varepsilon \Delta \varepsilon.$$

(1.61)

Here $\nu_B = \Omega_B / 2\pi$, $\nu_{B,0}$ is the nominal Brillouin frequency at some reference conditions, $\Delta T$ and $\Delta \varepsilon$ are variations from the nominal values of temperature and strain, respectively, and $C_T = 1$ MHz/°C and $C_\varepsilon \sim 0.05$ MHz/με for standard silica fibers. Figure 7 shows measurements of the Brillouin gain spectrum, its variations with temperature, and the extraction of the temperature coefficient $C_T$ from these measurements.
Using those dependencies, the estimation of the local Brillouin frequencies along a fiber under test may provide information on the local strain and temperature. In the majority of cases, this localization is achieved by measuring the position-dependent gain of Brillouin interactions as a function of the frequency offset. That procedure is usually referred to as Brillouin gain mapping. An example of a gain map is shown in Figure 8.
Various implementations of SBS-based sensors differ mainly in the method by which the Brillouin interaction is localized. They can be categorized into two major classes: time domain techniques and correlation domain techniques. Despite their differences, the merit of all distributed sensing methods is quantified by the following metrics:

1. Sensing range;
2. Spatial resolution;
3. Experimental uncertainty in the measurement of the local Brillouin shift;
4. Acquisition time.

In the following sections I will survey some of the developments in each category and compare the different realizations with respect to the above metrics.

1.4.1 Time Domain sensing methods

In Brillouin optical time domain analysis (B-OTDA), a pulsed pump wave amplifies a CW probe whose intensity is monitored as a function of time [24]. Since the gain observed at each instant can be traced back to a specific fiber location, local interactions can be resolved. The temporal analysis is carried out for a number of frequencies in the vicinity of $\nu_B$, resulting in a Brillouin gain map, as seen in Figure 8.

B-OTDA was first proposed in academic research 25 years ago [22]. In the past 15 years, it was implemented commercially in a variety of applications: pipeline integrity monitoring, subsea monitoring of umbilical cords in rigs, electrical power line cable monitoring and more. A typical commercial B-OTDA setup can reach a sensing range of tens of kilometers, with meter-scale spatial resolution. Because a single pulse can scan the entire fiber, this method is highly suitable wherever long range measurements are required, with relatively short acquisition durations. For example, a commercial interrogator can scan a 50 km long fiber in under 10 minutes, with a standard deviation of frequency errors of 1 MHz, corresponding to sensitivity of 1 °C or 20 $\mu$ε [23]. The spatial resolution of such instruments is typically 2-3 meters.
The main limitation on B-OTDA sensing range is the accumulated loss of pump power as it propagates along the fiber. It stems from linear losses (about 0.2 dB/km) and from power transfer to the probe, a phenomenon known as pump depletion. See ref. [25] for a detailed analysis on the performance and limitations of long-reach B-OTDAs. The loss of power can be mitigated using in-line amplification of the pump. Recent publications report the use of Raman amplifiers to extend the range of B-OTDA to 120 km with 2 m resolution [26, 27]. Sensing range of 325 km was demonstrated using cascaded repeaters [28].

In terms of acquisition time, a B-OTDA setup is mainly limited by two factors: the round trip propagation time through the fiber under test and the number of repetitions necessary to achieve the required measurement error. It can be quantified by the following simple equation:

\[
T_{\text{map}} > N_\nu \times N_{\text{avg}} \times T_D,
\]

(1.62)

where \(N_\nu\) is the number of frequency offset values \(\nu\) used to map out the SBS gain spectra, \(N_{\text{avg}}\) is the number of repetitions taken and \(T_D = 2L/v_g\) is the round-trip propagation time for a fiber of length \(L\). Note that the length of the fiber under test affects both \(T_D\) and the SNR of the detected response [25], and thereby the number of necessary averages.

The lower bound on the acquisition time given in eq. (1.62) can be elevated by scanning only one frequency offset \((N_\nu = 1)\) as described in ref. [29]. In that work, a single \(\nu\) was constantly sampled and the gain relative to the nominal value is monitored. Careful choice of \(\nu\) on the spectral slope of the SBS Lorenzian gain line, near the point of half maximum, enables dynamic sensing of small mechanical vibrations. Measurement rates of few hundred Hz along 85 m of fiber where demonstrated [29]. In a more recent work, the method was further improved in speed and in resolution to rates higher than 10 kHz with resolution of 2 cm [30]. In a different approach, the number of averages \(N_{\text{avg}}\) can be reduced by using coding techniques, effectively launching several pump pulses within \(T_D\), as reported in ref. [31]. Sensing over 120 km was achieved.
Spatial resolution of standard B-OTDA schemes is governed by the length of the pulsed pump. It can be derived from eq. (1.54) that the excitation of the acoustic field by pulses shorter than the acoustic life time $\tau$ will result in under-saturated acoustic fields, and thus limit the gain of the probe [32, 33]. Further, it can be also shown that pulses shorter than $\tau$ broaden the natural linewidth of $\Gamma_{\beta} \approx 2\pi \times 30 \text{MHz}$, thus making the identification of the Brillouin frequency shift more difficult [32, 33].

Several methods where proposed in recent years to circumvent this limitation while maintaining the basic concept of B-OTDA [9, 34]. One common idea underlying several of the proposed methods is the pre-excitation of the acoustic field. The interrogating pulse itself can be a short and intense ('bright pulse', see [35]) preceded by a longer, weaker pump pedestal. The added gain of the short stronger pulse relates to a pre-existing acoustic wave whose amplitude is already established at some nonzero value. For that reason, the transition time of probe gain is not restricted by the acoustic life time. Alternatively, the interrogating pulse can be replaced by a ‘dark pulse’ – a short segment with zero optical power [36], or a pi phase shift (‘pi pulse, see [37, 38]). In all cases, cm scale resolution was successfully reached, breaking the inherent 1 m limitation of standard B-OTDAs. The drawback of those methods is that the interrogating pulse, albeit short, slightly alters the acoustic field. This causes an ‘echo’ of slight ambiguity between neighboring locations.

The differential pulse pair (DPP) B-OTDA technique [5, 39] is another common method of high resolution, time domain Brillouin sensing. It relies on the concept of pre-excitation, but uses two relatively long pulses instead of one, launched sequentially into the fiber where the duration of one pulse is designed to be slightly shorter than that of the other. The measured gain of the shorter pulse is subtracted from the gain of the longer one via post-processing. The result of the subtraction corresponds to a highly localized Brillouin interaction whose spatial extent relates to the difference between the widths of the pulses, limited only by instrumentation and SNR considerations. Figure 9 plots the SBS signals obtained using an 8 ns-long and an 8.2 ns-long pulse pair, and the difference between the traces along 2 km of fiber [5]. A 2 cm-long segment towards the output end of the pump wave was locally heated. The hot spot was properly recognized in the measurement. The standard deviation in
the estimate of the local Brillouin frequency shift was about 2 MHz. Based on a similar concept, differentiation of the output signal was also shown to provide high resolution sensing [40].

![Figure 9: SBS signals from a pulse pair of 8/8.2 ns durations, and their difference trace, in a 2 km long fiber under test [5].](image)

1.4.2 Correlation domain sensing methods

In all time domain techniques discussed in the preceding section, a transient acoustic field was interrogated. High resolution requirements push the temporal extent on the acoustic grating to be as short as possible. The short signals mandate very high bandwidth modulation and detection. High rate detection is disadvantageous in terms of cost and associated measurement noise, such as shot noise and additive thermal noise, whose variances scale with bandwidth.

In Brillouin optical correlation domain analysis (B-OCDA), a localized acoustic field is held constant in time by constant-magnitude optical waves. The gain induced by the Brillouin interaction can be measured using arbitrarily low detection bandwidths, breaking the trade-off between high resolution and detection rate. A correlation based approach enables very high resolution, down to the mm scale, while maintaining a comparatively simple realization.
In order to understand how constant-magnitude waves can induce a localized acoustic field, we turn our attention to eq. (1.54), repeated here for convenience:

\[
\rho(z,t) = jg_1 \int_{-\infty}^{r} e^{-T_x(t-t')} A_1(z,t') A_2^*(z,t') dt'
\]  

(1.63)

To gain more intuition, we simplify expression (1.63) by assuming that the optical fields are unaffected by the Brillouin interaction. This assumption is relevant to B-OCDA setups, since the acoustic field is typically restricted to very narrow peaks, thus limiting the effective interaction lengths and the accumulated probe gain. Under this assumption, the non-homogeneous term on the right hand side of eq. (1.44) and (1.45) vanishes, and the solution to the propagating optical fields under a given boundary condition is simply:

\[
A_1(t,z) = A_{1,0} \left( t - \frac{z}{v_g} \right)
\]  

(1.64)

\[
A_2(t,z) = A_{2,0} \left( t + \frac{z-L}{v_g} \right)
\]  

(1.65)

where \(A_{1,0}(t)\) and \(A_{2,0}(t)\) are the complex envelopes of the optical waves injected into the fiber ports at \(z=0\) and \(z=L\) respectively. Equation (1.63) then reduces to:

\[
\rho(z,t) = jg_1 \int_{-\infty}^{r} e^{-T_x(t-t')} A_{1,0} \left( t' - \frac{z}{v_g} \right) A_{2,0}^* \left( t' + \frac{z-L}{v_g} \right) dt'
\]  

(1.66)

It is evident from eq. (1.66) that there is a close relation between the acoustic field amplitude and the temporal cross-correlation between the input optical amplitudes \(A_{1,0}(t)\) and \(A_{2,0}(t)\). Although the integral in (1.66) is weighed by an exponential window, it is nevertheless possible to manipulate the spatial extent of the acoustic field by using optical fields with specifically designed correlation properties.

The group of prof. Hotate at the University of Tokyo was the first to propose the use of modulated CW waves in a high resolution Brillouin sensing setup, a
technique that came to be known as B-OCDA. In their initial demonstration [41], both pump and probe were frequency modulated by a common sine wave. The cross-correlation of such modulated waves has distinct, isolated and periodic peaks, ('correlation peaks'). The spatial resolution is governed by the width of a single correlation peak, given by [6]:

\[
\Delta z = \frac{v_g \Gamma_g}{2\pi f_m \Delta f}.
\]

Here \( f_m \) is the modulation frequency and \( \Delta f \) is the frequency modulation range of the optical waves. The maximum range of unambiguous measurements in this configuration is the separation between adjacent peaks, given by [6]:

\[
d_m = \frac{v_g}{2f_m}.
\]

Since the range of unambiguous measurements is inversely proportional to the modulation frequency, ideally \( f_m \) would kept as slow as possible. However, \( \Delta z \) is also inversely proportional to \( f_m \), which means that low-rate modulation will also cause lower spatial resolution thus imposing a strict trade-off between resolution and range. This restriction can be somewhat relaxed by using the second degree of freedom, the frequency modulation amplitude \( \Delta f \), but only up to a certain extent – large frequency modulation spans cause unwanted amplitude variations. In a work published in 2000, Hotate and co-authors demonstrated B-OCDA sensing with 37 cm spatial resolution and with an unambiguous range of about 10 m. The setup was later extended to reach a very high resolution of 1.6 mm [6], a figure unattainable by any time-domain Brillouin sensing setup to-date. The measurement range was since extended using more elaborate frequency modulation profiles [42, 43], and reached 34 meters with few mm spatial resolution, or 500 m with 12 cm resolution [44].

1.5 Dynamic Brillouin gratings

The acoustic wave excited during an SBS interaction can be described as a Bragg grating, tuned to the wavelength of the pump (see section 1.3.2). Since the grating is non-permanent and may be turned on and off, or even moved to a desired
location, it is often referred to as a dynamic Brillouin grating (DBG). Use of polarization maintaining (PM) fibers allows one to excite a DBG using two pump waves polarized along one axis, while probing the DBG with waves polarized along the orthogonal axis, as illustrated in Figure 10.

![Figure 10: Polarizations, optical frequencies and directions of propagation of the optical waves used to generate and probe a DBG [45].](image)

In the arrangement depicted in Figure 10, two pump waves, pump 1 and pump 2 of respective frequencies $\nu_{x,1}$ and $\nu_{x,2}$, are separated in frequency by the Brillouin frequency shift: $\nu_{x,2} - \nu_{x,1} = \frac{\Omega_\beta}{2\pi} \equiv \nu_\beta$. The two pump waves counter propagate through a PM fiber while being polarized along the ‘slow’, or $\hat{x}$, axis of the fiber. A third wave, called the signal wave, is launched into the fiber along the ‘fast’ or $\hat{y}$ axis of the PM fiber. Its wavelength is tuned to match the wavelength of Pump 2 in the fiber medium and thus meet the Bragg condition (Eq. (1.25)) of the DBG. Due to the birefringence of PM fibers, the optical frequency of the signal $\nu_y$, differs from that of pump 2 by [46]:

$$\Delta \nu \equiv \nu_y - \nu_{x,2} = \nu_{x,2} \frac{n_x - n_y}{n_y}$$ (1.69)
Where \( n_x \) and \( n_y \) are the effective refractive indices of \( \hat{x} \) and \( \hat{y} \) polarized waves in the PM fiber, respectively. The signal wave is reflected by the DBG, and the reflected wave is Doppler shifted to frequency \( \nu_{\gamma}^{\text{R}} \) such that \( \nu_{\gamma}^{\text{R}} = \nu_{\gamma}^{\text{S}} - \nu_{\gamma} \). This arrangement allows for effective separation between the ‘writing’ pumps and the reading signals using both polarization discrimination and spectral filtering.

DBGs over PM fibers provide a promising sensing platform, as both \( \nu_{\gamma} \) and the fiber birefringence vary with strain and temperature [47-49]. Cascaded DBGs were used in microwave-photonic filters [50]. DBGs can also represent 'movable mirrors' for the variable all-optical delay of interrogating signal waves [51-53]. The effective delay of broadband waveforms requires that the DBGs are spatially confined: cm-scale gratings are necessary to accommodate Gb/s data. In addition, the magnitude of the index variations must be temporally stable. The localization schemes described in section 1.4 are also applicable for this purpose, with roughly the same limitations. In [51], variable delays were demonstrated using DBGs written by pulsed pump waves. However, the inscription of short gratings was restricted by the relatively long acoustic lifetime which corresponds to meter-scale spatial extent, similar to the limitation in B-OTDA schemes. Furthermore, pulsed DBGs are inherently transient and must be refreshed periodically.

Localized and stationary DBGs were introduced using correlation domain techniques, through synchronized frequency modulation (FM) of two continuous pump waves, nominally detuned in frequency by \( \nu_{\gamma} \) [52]. However, this scheme required the synchronized frequency modulation of the signal wave as well [52]. In another previous demonstration, a strain gradient was introduced along the PM fiber in order to achieve localization [53]. A technique to localize a stationary DBG is presented and demonstrated in Chapter 3, along with its application in the variable delay of broadband signals.

1.6 Objectives

The objectives of the presented work are as follows:
1. Expanding the capabilities of B-OCDA to arbitrarily long range of unambiguous measurements. The main limitation to the range of B-OCDA setups is the relatively short separation between adjacent correlation peaks. In this work, radar-inspired modulation techniques are utilized to form correlation functions with unboundedly long periods, thus lifting the trade-off between resolution and range mentioned in section 1.4.2.

2. Reduction of acquisition times by the simultaneous interrogation of multiple correlation peaks. Separation of different gain events is done via time domain analysis or by post processing.

3. Application of the distributed Brillouin sensing techniques to structural monitoring of composite materials with embedded optical fibers, under various mechanical conditions.

4. The study and realization of DBGs as 'movable mirrors' for the variable all-optical delay of wideband waveforms.

5. The study of forward SBS as an underlying mechanism for the analysis of media outside the outer boundary of the fiber cladding.
2 SBS Localization Via Phase Coding

2.1 Phase coded SBS – intuitive introduction

Referring back to Eq. (1.66), we note again that the acoustic field, and the SBS gain along with it, scales with the temporal integration of the product of the two optical fields, or in other words – their cross-correlation. Using this property, SBS localization can be achieved by the joint random modulation of the optical waves, as will be described in this section.

Consider a case where both pump and signal waves are jointly phase-modulated by a common binary pseudo random binary sequence (PRBS), whose symbol duration $T_p$ is much shorter than the acoustic life-time $\tau$. The modulation phase within each symbol assumes a value of either 0 or $\pi$, with equal probabilities, as illustrated in Figure 11a. Suppose at first that the phase modulation is synchronized so that the two waves are of equal phases at their respective entry points. The instantaneous driving force for the SBS acoustic field generation in any given position is proportional to the product of the pump wave envelope and the complex conjugate of the signal wave envelope, at that location. We may therefore distinguish between the dynamics of the SBS-induced acoustic field in two different regions. In the vicinity of the fiber center the pump and signal are correlated, hence their phase difference is constant and the driving force for the acoustic field generation keeps a steady non-zero value (Figure 11b). Consequently, the acoustic field is allowed to build up to its steady state value (Figure 11c).

In all other locations, the driving force for the acoustic field is randomly alternating in sign on every symbol duration $T_p \ll \tau$. The acoustic field magnitude thus averages out to a zero expectation value, and the SBS interaction outside the correlation peak is largely inhibited. The location of the correlation peak can then be arbitrarily selected simply by introducing a time-delay to one the optical signals.

As will be shown in the next section, the spatial extent of the correlation peak is $\Delta z = v_s T_p / 2$, while the distance between adjacent peaks is $M_p v_s T_p / 2$ where $M_p$ is the length of the PRBS sequence. Since arbitrarily long sequences can be readily
generated, resolution and unambiguous range are decoupled, in contrast to traditional frequency modulation correlation-domain techniques [6].

\[
\begin{align*}
A_1(t, z = 0) &= A_2(t, z = L) = A(t) = A_0 \sum_n c_n \text{rect}\left(\frac{(t - n T_p)}{T_p}\right) \\
\end{align*}
\]  

\[\text{(2.1)}\]

Here, \(A_0\) denotes the constant magnitude of both waves and \(c_n\) is a PRBS. The resultant acoustic field is given in Eq. (1.66). Substituting (2.1) into (1.66), we have:

Figure 11: The principle of phase coded, localized SBS. a) Illustration of binary phase modulated Brillouin probe (top row) and pump (bottom row) waves. b) The instantaneous driving force for the generation of the SBS acoustic field. c) The magnitude of the resulting acoustic field, obtained by temporal integration over the driving force.

2.2 Phase coded SBS – mathematical description

2.2.1 General definition

Consider two co-modulated, counter-propagating pump and probe waves, as described in Equations (1.28) and (1.29), with input complex amplitudes:
\[
\rho(z,t) = jg_1 \int_{-\infty}^{t} e^{-\Gamma \left(t-t'\right)} A\left(t'-\frac{z}{v_g}\right) A^* \left[t'-\frac{z}{v_g} - \theta(z)\right] dt' \quad (2.2)
\]

The position-dependent temporal offset \( \theta(z) \) in Eq. (2.2) is defined as:
\[
\theta(z) = (2z - L)/v_g.
\]
Substituting (2.1) into (2.2), we find that when the signals are modulated by a sequence of rectangular pulses, the integral can be approximated by a discrete summation:
\[
\rho(t,z) \approx jg_1 |A_i| T_p \sum_{n=-\infty}^{n_o(z)} \exp\left[\frac{(n-n_o)T_p}{2\tau} \right] c_n c_{n-l}^* = jg_1 |A_i| T_p \tilde{R}_{n_o}(l_z) \quad (2.3)
\]

In Eq. (4), the following variables were introduced: \( l_z \) is the normalized, position-dependent lag between the sequences: \( l_z = \text{round} \left( \theta(z)/T \right) \), and \( n_o(t,z) \) is the number of the bit appearing in \( A_i \) at position \( z \) and time \( t \): \( n_o = \text{round} \left( \left( t-z/v_g \right)/T_p \right) \). Lastly, \( \tilde{R}_{n_o}(l_z) \) is an exponentially windowed auto-correlation function with a 'memory' of \( N_o = \text{round} \left( 2\tau/T_p \right) \) bits:
\[
\tilde{R}_{n_o}(l_z) = \sum_{n=-\infty}^{n_o(z)} \exp\left[-(n-n)/N_o\right] c_n c_{n-l}^* \quad (2.4)
\]

Note that in this approximate solution, the position dependence of \( \rho \) is governed by the discrete variable \( l_z \), which is constant in intervals of length \( \Delta z = v_g T_p / 2 \). This allows for a schematic 'discretization' of the acoustic field in the fiber into \( \Delta z \)-long bins, as illustrated in Figure 12. The magnitude of the acoustic field scales with the discrete auto-correlation function of the code \( c_n \). Noting that \( T_p \ll \tau \) and therefore \( N_o \gg 1 \), hence the following approximation can be made:
\[
\sum_{n=-\infty}^{1} \exp(n/N_o) = \frac{1}{\exp(1/N_o) - 1} \approx N_o \quad (2.5)
\]

Thus, the acoustic field is expected to show a peak at the \( l_z = 0 \) bin, where:
\[
\rho(t,l_z = 0) \approx jg_1 |A_i|^2 T_p N_o \quad (2.6)
\]
At the correlation peak the acoustic field is allowed to reach its maximum, steady-state, value. Elsewhere, the acoustic field depends on the temporal autocorrelation function of the code $c_n$.

\[ \rho / \int |A_i|^2 \, dT \]

\[ l_z = -4 \quad l_z = -3 \quad l_z = -2 \quad l_z = -1 \quad l_z = 0 \quad l_z = 1 \quad l_z = 2 \quad l_z = 3 \quad l_z = 4 \]

\[ \hat{R}_{N_0} (0) \quad \hat{R}_{N_0} (-1) \quad \hat{R}_{N_0} (1) \quad \hat{R}_{N_0} (3) \]

Figure 12: Illustration of the acoustic field magnitude as a function of position along a fiber. The solution is approximated by a discretization of the fiber into $\Delta z$-long bins.

### 2.2.2 Optical Signal-to-Noise Ratio Calculations

A good measure of the effectiveness of confinement is the optical signal-to-noise ratio (OSNR) of waves reflected by the acoustic grating. For that purpose, the signal is defined as the reflectance of the correlation peak, while the noise is the accumulated undesired reflectance of the acoustic waves along the rest of the fiber. Note that reflectance scales with $|\rho|^2$. From Eq. (2.3) we learn that while the correlation peak magnitude is constant in time and independent of the choice of code, the side-lobes are fluctuating and significantly affected by the specific code $c_n$.

However, a general expression for the expectation value of the OSNR of reflected waves may be given. In the following it is assumed that for any $p \neq k$, $\hat{R}_{N_0} (k)$ and $\hat{R}_{N_0} (p)$ are statistically independent of each other, therefore:

\[ E[\hat{R}_{N_0} (k)\hat{R}_{N_0}^* (p)] = 0 \quad (2.7) \]
We also assume that the expectation value $E \left( |\tilde{R}_{N_0}(k)|^2 \right)$ is the same for any $k \neq 0$. Given these considerations, the expectation value for the OSNR of reflected signals is:

$$\text{OSNR} = \frac{E \left( |\tilde{R}_{N_0}(0)|^2 \right)}{\sum_{l_z \neq 0} E \left( |\tilde{R}_{N_0}(l_z)|^2 \right)} = \frac{N_0^2}{N_{\text{bins}}} \sigma_R^2$$

(2.8)

The summation in (2.8) is taken for all $l_z$ that cover the length of the fiber $L$, $N_{\text{bins}}$ is the number of discrete bins of width $\Delta z$ in the fiber: $N_{\text{bins}} = \text{round}(L/l_z)$, and $\sigma_R^2 = E \left( |\tilde{R}_{N_0}(l_z)|^2 \right)$ is the variance of the exponentially-weighted correlation at off-peak positions. We note that the signal power will be independent of the code, as long as the code is of constant magnitude. We also note that the noise will scale with the length of the fiber, and can be highly influenced by the choice of code through the variance of its off-peak correlation $\sigma_R^2$.

In this work two classes of codes were studied and examined: PRBS sequences and a class of sequences called perfect Golomb codes.

2.2.2.1 PRBS codes

A PRBS code is defined here as a series $c_n$ in which each element assumes a value either 1 or -1, with equal probabilities and is statistically independent from any other element in the series:

$$P(c_n = 1) = P(c_n = -1) = 1/2,$$

(2.9)

$$E \left[ c_n c_p \right] = \delta_{k,p},$$

(2.10)

where $\delta_{k,p}$ is the Kronecker delta. The variance of the correlation side-lobe $\sigma_R^2$ is calculated as follows:
\[ \sigma^2_R \equiv E \left\{ \left| \hat{R}_{N_v} (l) \right|^2 \right\} \]
\[ = \sum_{n_{m(t,x)}} \sum_{m_{l(t,x)}} \exp \left\{ -\left( m_0 - m \right)/N_v \right\} \exp \left\{ -\left( n_0 - n \right)/N_v \right\} E \left[ c_m c_n^* \right], \quad (2.11) \]
\[ = \sum_{n_{m(t,x)}} \exp \left\{ -2\left( n_0 - n \right)/N_v \right\} \approx \frac{N_v}{2} \]

where the identity (2.5) was used. Substituting into (2.8), the expectation value for the OSNR of PRBS coded gratings is found to be:

\[ \text{OSNR(PRBS)} = \frac{2N_v}{N_{\text{bins}}} = \frac{2\tau v}{L} \quad (2.12) \]

It is important to note in (2.12) that the OSNR is not expected to depend on the phase modulation rate (or spatial resolution). It is given by the length of the fiber and cannot be altered. An intuitive reasoning can be made with the following argument: while increasing modulation rate reduces noise by decreasing the variance of the correlation side-lobes, it also reduces the width of the correlation peak and thereby attenuates the signal power.

**2.2.2.2 Golomb codes**

The OSNR value in (2.12) may be insufficient in many cases, especially in scenarios where averaging over multiple repetitions of a measurement is not possible. In order to improve the OSNR, \( \sigma^2_R \) must be reduced. Fortunately, related problems had been addressed in radar theory. For example, Prof. Solomon Golomb of the Univ. of Southern California (USC) designed a category of so-called prefect codes, having the following useful properties [54]: let \( a_n \) denote a perfect Golomb code, repeating with a period of \( M_p \) bits. Its cyclic auto-correlation:

\[ R_{M_p} (l) = \sum_{n=M_p \cdot M_p}^{n_0} a_n a_{n-l}^* \quad (2.13) \]

is exactly zero for all \( l \neq 0 \) and all \( n_0 \) [54]. Since \( \hat{R}_{N_v} \) represents an exponentially windowed auto-correlation which is not identical to \( R_{M_p} \), we do not expect \( \sigma^2_R \) to vanish entirely. Nevertheless, it is anticipated that Golomb codes would improve the
OSNR by a considerable amount. In the next section, this hypothesis is examined through numerical simulations.

### 2.3 Simulations

We first turn to the numerical evaluation of $\sigma_R^2$ for Golomb codes, in order to formulate a more quantitative measure of their expected performance in the confinement of SBS. To that end, we numerically calculate the sum in Eq. (2.4) as a function of $N_0$ and for Golomb codes of various lengths $M_p$. The results are shown in Figure 13.

![Figure 13: Simulated $\sigma_R^2$ values as a function of $N_0$. Golomb codes of length 63, 83, 103 and 127 were used in the simulation. The results of PRBS modulation is shown as well for comparison.](image)

Interestingly, the values of $\sigma_R^2$ obtained for Golomb codes are bound by an asymptotic value for $N_0 > M_p$, in contrast to PRBS where $\sigma_R^2$ scales with $N_0$ indefinitely. The asymptotic $\sigma_R^2$ values of Golomb codes of several lengths are listed in Table 1.

The acoustic fields arising from phase coded optical pump waves can be evaluated using numeric integration of Eq. (2.2), with appropriate boundary conditions. The results of such simulations are shown in Figure 14. The following parameters were taken in the simulation: $|A_0|^2 = 1\ W$, $\tau = 5.3\ ns$, $L = 1\ m$, and $T_p = 200\ ps$, which corresponds to $N_0 = 53$.  

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Table 1: Asymptotic $\sigma_R^2$ values for Golomb codes of various lengths.

<table>
<thead>
<tr>
<th>$M_p$</th>
<th>Asymptotic $\sigma_R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>5.3</td>
</tr>
<tr>
<td>83</td>
<td>7</td>
</tr>
<tr>
<td>103</td>
<td>8.7</td>
</tr>
<tr>
<td>127</td>
<td>10.6</td>
</tr>
</tbody>
</table>

Figure 14: Acoustic field magnitude $|p(z, t)|$ in units of kg/m$^3$. The acoustic field is driven by optical pump and probe waves, phase modulated by PRBS (top) and by a perfect Golomb code (bottom, $M_p = 83$). The symbol duration is $T_p = 200$ ps in both simulations.
In Figure 14, the acoustic field magnitude $|\rho|$ is plotted as a function of position and time. It is driven by phase modulated pump and signal waves, with step-function intensities. As can be seen, the acoustic field starts to build up at specific locations and instances where the pump waves meet. However, only at the center of the phases of the pump and signal are correlated. Therefore only at that location the acoustic field excitation is monotonous and allows the acoustic wave to reach a constant steady state. In all other positions the pump and signal are not correlated (only one correlation peak is present in the fiber). The acoustic field oscillates about an expectation value of zero magnitude, and it is largely suppressed compared to its magnitude the correlation peak.

It is important to note that the correlation side-lobes of a PRBS driven acoustic field are higher than those of a field induced by Golomb coded pumps. This is of course expected, given the lower $\sigma^2_{R}$ values of Golomb codes: for the parameters of the simulation, $\sigma^2_{R} = 7$ for the Golomb code and $\sigma^2_{R} = 26$ for PRBS. In both cases, the instantaneous magnitude of the acoustic field at off-correlation locations is fluctuating with non-zero variance. These residual off-peak stimulations add to the measurement noise in phase-coded SBS processes.

2.4 Summary

Based on the considerations discussed in this chapter, the following conclusions can be made with respect to SBS driven by phase modulated optical waves:

1. The acoustic field generated by the waves is effectively confined to discrete correlation peaks, where it reaches its saturation value and it is constant in time.

2. The width of the correlation peak is determined only by the modulation symbol duration $T_p$.

3. The separation between adjacent correlation peaks is governed by $T_p$ and by the length of the sequence $M_p$. The constraints linking
between resolution and range of unambiguous measurements are therefore largely removed.

4. Correlation side-lobes generate unwanted reflections that contribute noise (referred to hereunder as coding noise). Use of perfect Golomb codes is found to reduce the variance of the acoustic field spatial side-lobes in comparison to PRBS phase coding.

5. The side-lobe variance is highly dependent on $T_p$. A shorter $T_p$ reduces side-lobes, but also narrows the spatial extent of the correlation peak and therefore reduces the intended probe gain. The symbol duration of the phase code is therefore a key parameter that must be tailored specifically to the desired application.

In the following chapters, phase coded SBS interactions are studied in two main contexts: DBG-based delay lines and distributed sensing. In static sensing applications, the averaging of the output probe trace over many repetitions of measurements is typically possible, and coding noise may be reduced accordingly. In fact, as will be discussed in Chapter 4, there are some cases in which other noise mechanisms dominate over coding noise and it may be neglected altogether. In contrast, when phase coded DBGs are used as ‘movable mirrors’ to delay real-time signals, averaging is irrelevant and coding noise must be reduced to a minimum. Golomb codes then prove to be a major factor in enabling comparatively long delays. For further details, see the next Chapter 3.
3 Dynamic Gratings Based on Phase-Coded SBS

3.1 Introduction and motivation

A DBG based on phase-coded pump waves is a direct manifestation of the properties discussed in the previous chapter. An acoustic field driven by phase coded pumps is stationary and its spatial width and location can be controlled and switched within a few nano-seconds in an all-optical manner. The probing of phase-coded DBGs allows for a direct experimental validation of the quantitative properties of the induced acoustic waves, e.g. spatial extent of the correlation peak, side-lobe reflectivity etc. As will be shown below, phase-coded DBGs may be used in all-optical variable group delay of data. The stimulation and addressing of phase-coded DBGs is therefore of both fundamental and applicative interest.

In the analysis, simulation and experiments to follow we consider the case depicted in Section 1.5 (see Figure 10), where two pump waves, detuned in frequency by $\nu_B$, are polarized along the $\hat{x}$ axis of a PM fiber. The acoustic gratings inscribed by the pumps are probed by a third wave, polarized along the $\hat{y}$ axis. The center frequency of the probe wave is set to match the condition of maximum reflectivity of the DBG (given in Eq. (1.69)).

The figures of merit of a DBG, when used as a 'movable mirror' for the variable delay of optical communication data, for example, can be quantified by the following metrics:

1. Supported bandwidth of delayed probe signals: $\Delta f, \Delta \lambda$.
2. Extent of time delay variations $T_0$.
3. OSNR of reflected signals.
4. Reflectivity $R$.

These four metrics depend on the modulation parameters of the pump waves, and trade-offs typically exist among them.
3.2 Phase coded DBGs – theoretical expectations

We shall now analyze the above performance metrics, and their dependence on the properties of the driving pump waves and of the fiber under test. The parameters of the analysis are:

1. Phase code symbol duration $T_p$.
2. Phase modulation sequence length $M_p$.
3. Choice of code.
4. Optical power of pump waves $|A_0|^2$.
5. Fiber length $L$.

In the analysis below, it is assumed that the interrogating signal wave does not affect the DBG itself, so that the DBG acts as a linear reflector similar to a ‘moving FBG’. This assumption is valid as long as the DBG has a very low reflectivity. If this is case, the electrostrictive force induced by the interference pattern between the incident signal wave and the reflected wave is significantly lower than the force induced by the two pump waves.

3.2.1 Bandwidth

Consider a FBG with length $\Delta z$. The refractive index $\tilde{n}$ associated with the FBG is described by:

$$\tilde{n}(z) = \begin{cases} 
  n_0 + (\Delta n e^{i 2 \pi z / \Lambda} + \text{c.c.}) & 0 < z < \Delta z \\
  n_0 & \text{Elsewhere} \end{cases}.$$  \hspace{1cm} (3.1)

Here, $\Delta n$ is the amplitude of the refractive index change and $\Lambda$ is the period of the grating. If $|\Delta n| \ll \Lambda / \Delta z$ the grating is considered to be in the ‘weak reflectivity limit’ [55], and its reflection bandwidth is given by [55]:

$$\Delta \lambda = \frac{2 \lambda_0}{N_{\text{cycles}}}. \hspace{1cm} (3.2)$$

Here $\lambda_0 = 2 n_0 \Lambda$ is the wavelength of maximum reflectivity and $N_{\text{cycles}} = \Delta z / \Lambda$ is the number of spatial periods in the grating. The above value of $\Delta \lambda$ refers to the
wavelength difference between the first two zeros in the reflectivity spectrum, at both sides of the reflectivity maximum.

The reflectivity of a DBG varies with time. Nevertheless, the reflectivity spectrum can be determined based on similar considerations. The period of the DBG is \( \Lambda_{\text{DBG}} = \frac{\lambda_{p_2}}{2n_0} \), where \( \lambda_{p_2} = c / \nu_{p_2} \) is the wavelength of Pump 2. The bandwidth of a phase coded DBG is therefore:

\[
\Delta \lambda = \frac{\lambda_{p_2}}{n_0 \Delta z}.
\]

In terms of frequency, we have:

\[
\Delta f = \frac{\lambda_{p_2}}{n_0 \Delta z} \nu_{p_2} = \frac{v_g}{\Delta z} = \frac{2}{T_p}.
\]

This bandwidth corresponds to a rectangular grating profile. Solving (2.2) more accurately, we find that the spatial bins to which the fiber is divided are not rectangular, but rather triangular in shape. The spatial profile of the acoustic field is therefore of the form:

\[
\rho(t,z) \approx jg|A_0|^2 T_p \sum_{l} \text{tri} \left( \frac{z-l\Delta z}{\Delta z} \right) \tilde{R}_{n_0}(l),
\]

where the triangle function is defined as:

\[
\text{tri}(\zeta) = \begin{cases} 
1 - |\zeta|, & |\zeta| < 1 \\
0, & \text{Otherwise}
\end{cases}.
\]

The profile for the acoustic field at the correlation peak may be found by considering only the term \( l = 0 \) in the summation:

\[
\rho_{\text{correlation peak}} \approx jg|A_0|^2 T_p N \text{tri} \left( \frac{z}{\Delta z} \right).
\]
The reflectivity bandwidth of the DBG can be evaluated using the impulse response of reflection from the correlation peak. Consider an impulse input signal wave:

$$A_i(z,t) = A_{0i} \delta(z - tv_s). \quad (3.8)$$

The reflected wave would be the result of a convolution between the input signal and the acoustic field magnitude:

$$A_r(z,t) \approx j g_i A_s |A_0|^2 T_p N_0 \text{tri}\left( \frac{z + tv_s - L}{\Delta z} \right). \quad (3.9)$$

The power reflectivity is given by the magnitude squared of the Fourier transform of the output wave:

$$|H(j2\pi f)|^2 = |F\{A(z = 0,t)\}|^2 = \left( g_i A_s |A_0|^2 T_p N_0 \frac{\Delta z}{v_g} \right)^2 \text{sinc}^2 \left( \frac{\Delta z}{v_g} f \right). \quad (3.10)$$

The bandwidth of the response may be approximated as:

$$\Delta f \approx \frac{v_g}{\Delta z}, \quad (3.11)$$

reproducing the result of (3.4).

### 3.2.2 Maximum time delay

Time delay of reflected signals is given by the two-way time-of-flight from the signal input end of the fiber to the correlation peak and back. Minimum delay is zero. Maximum delay $T_D$ is achieved when the correlation peak is located at the far end of the fiber, in which case the delay is:

$$T_D = \frac{2L}{v_g}. \quad (3.12)$$

### 3.2.3 Optical signal-to-noise ratio

The general expression for the expectation value of the OSNR of the reflected signal, derived in section 2.2.2 is repeated here for convenience:
\[ \text{OSNR} = \frac{N_0^2}{N_{\text{bins}} \sigma_R^2}. \] (3.13)

It is interesting to note that for the case of PRBS coding, the OSNR is found to be dependent only on \( T_D \):

\[ \text{OSNR (PRBS)} = \frac{4 \tau}{T_D}. \] (3.14)

A tight tradeoff therefore prevails between OSNR and delay variations in a DBG written by PRBS-coded pumps. Their product is bound by \( 4 \tau \approx 20 \text{ ns} \), regardless of modulation rate. This limitation is relaxed when Golomb codes are used instead of PRBSs, however the inverse dependence of OSNR on delay remains regardless of the specific choice of code.

### 3.2.4 Reflectivity

The reflectivity of a DBG is defined as the maximum of its reflectivity spectrum. It can be calculated by the following relation [55]:

\[ R = \tanh \left( \frac{\pi}{2n_0} \Delta n N_{\text{cycles}} \right) \approx \left( \frac{\pi}{2n_0} \Delta n N_{\text{cycles}} \right)^2. \] (3.15)

Here, again, the ‘weak grating limit’ was assumed. Substituting the magnitude \( \Delta n \) from Eq. (1.24) yields:

\[ R \approx \left( \frac{\pi \gamma g_{1s}}{n_0 \rho_s \lambda \gamma} |A_o|^2 \Delta z \right)^2. \] (3.16)

### 3.2.5 Expected performance and figure of merit

In designing a phase coded DBG, all of the parameters mentioned above, e.g. \( T_p, \lambda_p, L, |A_o| \), must be taken under consideration in order to maximize the performance in terms of time delay, bandwidth, OSNR, and reflectivity. To summarize, the dependencies are brought in Table 2. We can immediately infer that increasing the modulation rate will broaden the supported bandwidth, as can be expected.
Table 2: Phase-coded DBG properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>$\Delta f = \frac{2}{T_p}$</td>
</tr>
<tr>
<td>Time delay</td>
<td>$T_D = \frac{2L}{V_g}$</td>
</tr>
<tr>
<td>OSNR</td>
<td>PRBS: $\frac{4\tau}{T_D}$, Golomb: $\frac{N_0^2}{N_{\text{bins}}\sigma_R^2}$</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>$R \approx \left( \frac{\pi^2g_1\tau}{n_0p_0^2</td>
</tr>
</tbody>
</table>

The length of sequence $M_p$ must be increased to ensure that only one correlation peak is generated in the fiber:

$$M_p > L / \Delta z .$$ (3.17)

Since $M_p$ is essentially unlimited, the available time delay can be extended by extending $L$, while maintaining high bandwidth. A commonly used figure of merit for delay line applications is the delay-bandwidth product. In this case, this product simply equals $N_{\text{bins}}$:

$$T_D \times \text{BW} = N_{\text{bins}} = L / \Delta z .$$ (3.18)

This figure of merit is only dictated by $M_p$. However, it does not take into account the noise generated by undesired reflections. As a ‘rule of thumb’, we restrict our discussion to OSNR levels of 13 dB or higher, which are typically required to maintain sufficiently low error ratios in optical communication. Under these restrictions, the maximum time delay allowed by PRBS coding must be below 1 ns. Since the OSNR is independent of modulation rate in the case of PRBS coding, the rate may be increased in order to achieve higher bandwidths. However, bandwidth is restricted by both instrumentation and by the reflectivity, which is inversely dependent on the modulation rate.

Since the off-peak reflectivity of a Golomb code $\sigma_0^2$ saturates at high values of $N_0$ (see Table 1), it is possible to increase the delay-bandwidth product by decreasing
For example, use of a Golomb code with $M_p = 63$, subject to the restriction of OSNR $>13$ dB, provides:

$$T_b \times BW = \frac{1}{25} \left( \frac{\tau}{T_p} \right)^2. \quad (3.19)$$

Given a modulation rate of $T_p = 100 \text{ ps}$, the delay bandwidth product of Golomb coded DBGs can be as high as 100, a significant improvement with respect to the performance of SBS slow light-based realizations, for example [56].

### 3.3 Simulations

Numerical simulations were carried out in two stages. First, the acoustic magnitude $\rho(z, t)$ was calculated as described in Section 2.3, with the following parameters: $\tau = 5.3 \text{ ns}$, $L = 0.9 \text{ m}$ ($T_d = 9 \text{ ns}$), $|A_0|^2 = 1 \text{ W}$ and $T = 100 \text{ ps}$ ($N_0 = 106$). A comparison between the spatial profiles of the acoustic field magnitudes generated by PRBS and Golomb coded pumps, at $t = 10\tau$, is shown in Figure 15.

![Figure 15: Simulated acoustic field magnitude as a function of position, for DBGs written by PRBS coded pumps (red) and by pumps modulated with a perfect Golomb code (blue), at t=10\tau [57].](image)

In the second simulation step, the DBGs corresponding to $\rho(t, z)$ were probed by signal waves, modulated by binary phase-shift-keying (PSK) PRBS data at 5 Gb/s. The intended signal power was calculated by artificially nullifying all off-peak DBGs
and the noise power was calculated by removing the correlation peak. The OSNR values were averaged over multiple realizations. The expectation value of the OSNR for PRBS pump modulation was only 2.1, in agreement with the prediction of 2.35 in Eq. (3.14). In contrast, the Golomb code improved the OSNR by an order of magnitude to 20.5, meeting the minimum requirement. The OSNR prediction of Eq. (3.13) with $\sigma_R^2 = 5.3$ and $N_{\text{bins}} = 90$ is an OSNR of 23.5, which agrees well with the simulation. The results suggest that DBGs over PM fibers could provide a delay-bandwidth product of 45 while retaining sufficient signal integrity.

Figure 16 shows the simulated eye diagrams of the delayed 5Gb/s PSK waveforms, reflected from DBGs induced by PRBS-coded pumps (top) and Golomb-coded pumps (bottom). Golomb coding leads to an open eye diagram whereas PRBS coding results in a nearly closed diagram.

![Eye Diagrams](image)

Figure 16: Simulated eye diagrams of 5 Gb/s PSK data streams, delayed by DBGs induced along a 90 cm-long PM fiber. Top: PRBS-coded pumps. Bottom: Pumps modulated by a perfect Golomb code ($M_p = 63$). The coding symbol duration was 100 ps in both simulations [57].

3.4 Experiments

Two sets of experiments were performed in order to examine phase-coded DBGs. In the first set, PRBS codes were used to evaluate the localization and frequency response of the DBG. Coding noise was reduced using averaging over multiple repetitions of each experiment. In the second set, emphasis was given to side-lobe
reflectivity and OSNR, and the variable true time delay of broadband signals was demonstrated.

3.4.1 Experimental setup #1 – multiple repetitions

Figure 17 illustrates the experimental setup that was used for the generation and characterization of DBGs, driven by random phase modulation of both pump waves. A single distributed feed-back (DFB) laser diode of frequency \( \omega_{p1} \) was used to generate both pump waves. The output of the pumps DFB was modulated by an electro-optic phase modulator (EOM), which was driven by a PRBS generator. The output peak-to-peak voltage of the PRBS generator was adjusted to match \( V_e \) of the EOM (~ 3.7V), and its clock rate was controlled by a microwave generator. The modulated DFB light was split in two arms. Light in one path was amplified by an erbium-doped fiber amplifier (EDFA) to \( |A_1|^2 = 200 \text{ mW} \), polarized along the \( \hat{x} \) principal axis of a PM fiber, and launched into a section of specialty PM fiber under test (FUT) as pump 1. Light in the other arm was modulated by second EOM, which was driven by a sine wave of frequency \( \Omega \) from a second microwave generator. The EOM in this arm was biased to suppress the optical carrier at \( \omega_{p1} \). The upper modulation sideband of frequency \( \omega_{p2} = \omega_{p1} + \Omega \) was selected by a narrow-band FBG, amplified by a second EDFA to \( |A_2|^2 = 200 \text{ mW} \) and launched along the \( \hat{x} \) axis from the opposite end of the FUT as pump 2. The residual carrier wave and the other modulation sideband of pump 2 were rejected by the FBG.

Arbitrary locations along the fiber under test were addressed as follows. The joint phase modulation introduces multiple correlation peaks along the fiber ring that encompasses the pump 1 branch, the pump 2 branch, and the FUT. The peaks are separated by \( M_{\rho} \nu_s T_p / 2 \), hence the positions of all peaks, except for the central one, vary with \( T_p \). A path imbalance was deliberately inserted along the signal branch, so that an off-centered correlation peak was in overlap with the FUT. The position of this peak could be scanned through slight changes to the PRBS symbol duration: Variation of the clock rate by 1 MHz corresponded to an offset of the correlation peak position.
by about 1 m. At the same time, the code length was sufficiently long to guarantee that only one peak was introduced within the FUT.

Figure 17: Experimental setup for the generation and characterization of DBGs, generated by phase-modulated pump waves. PC: polarization controller.

Light from a second DFB laser was used to generate the readout signal wave. The optical frequency of the signal DFB was adjusted via temperature and current control to match the frequency of maximum reflectivity of the acoustic dynamic grating along the \( \hat{y} \) principal axis, given in Eq. (1.69). In the FUTs used in this experiment, \( \Delta n \) was on the order of \( 5 \times 10^{-4} \), corresponding to \( \Delta \nu \approx 80 \text{ GHz} \). Some of the measurements relied on continuous wave readout signals, whereas in other experiments a third EOM was used to generate readout pulses or sine-wave modulation, as necessary. The readout signal wave was polarized along the \( \hat{y} \) principal axis of the PM FUT, and was launched to probe the DBGs. The reflected signal of frequency \( \nu_r = \nu_s + \Delta \nu \) was filtered by another FBG, amplified and measured using either a low-bandwidth or a high-bandwidth detector, depending on the specific experiment.
The localization of the dynamic gratings was first demonstrated using periodic, isolated readout pulses that were 260 ps long. Figure 18 shows the detected power of the reflected signal pulses as a function of time. Maximum reflectivity was achieved for $\Omega = \Omega_0 = 2\pi \times 10.318$ GHz and $\Delta \nu = 84$ GHz. When CW pumps were used (PRBS generator switched off), a distributed reflection of the readout pulses was observed, with a duration of $2L / v_g \sim 10$ ns (see curve (a)). The distributed reflection suggests that an extended, nearly uniform dynamic grating had been generated along the entire length of the FUT, the amplitude change being simply accounted to a fluctuating birefringence. Curves (b) and (c) show the temporal profiles of the reflected readout pulses with the PRBS phase modulation of the pumps waves switched on, using $T_p = 1$ ns and 167 ps respectively. A reflection from a localized dynamic grating is evident. The reflection from the shorter grating written with $T_p = 167$ ps is weaker, as expected. All traces in Figure 18 were averaged over 20 repetitions of the readout pulses. Figure 19 compares the temporal profile of the input pulse power and that of the reflected pulse, using $T_p = 167$ ps. The full width at half maximum of the reflected pulses was broadened by 40%, in line with the expected, distributed reflection from the grating length.

Figure 18: Reflected waveforms from DBGs, interrogated with 260 ps long, isolated periodic readout pulses. a) – Grating written by continuous wave pumps. b) The phases of both pump waves was modulated by a PRBS, $T_p = 1$ ns. c) Same as b), $T_p = 167$ ps.
The frequency response of the dynamic gratings reflections was characterized using a vector network analyzer (VNA). An EOM in the readout signal path was driven by a swept sine-wave from the VNA output port, and the detected reflection was analyzed using the input port of the instrument. Figure 20 shows the relative transfer function of the reflected signal power, as a function of the offset between the readout frequency and $\omega_s$ of maximum reflectivity. As expected, the dynamic grating becomes shorter with an increase of the PRBS rate, and the reflection bandwidth broadens accordingly. The bandwidths of readout signals that can be accommodated by the dynamic gratings are on the order of the clock rates of the PRBS pump modulation, as expected (see Eq. (3.4)). The relatively high reflectivity at the low frequencies (offset < 200 MHz) is due to a residual extended grating, spanning the entire length of the FUT: the CW components of the pump waves could not be entirely suppressed due to a small mismatch between the driving voltage magnitude of the phase EOM and its $V_x$ value.
The variable delay of isolated readout pulses was demonstrated along a 100 m long FUT (\(\nu = 10.87 \text{ GHz}, \ \Delta \nu = 57 \text{ GHz}\)). In this experiment, a PRBS of length \(M_p = 2^{10} - 1\) was used in the modulation of both pumps, and a fiber delay imbalance was added to the path of pump 1 so that the 10th-order correlation peak was scanned along the FUT. Figure 21 shows the relative delay of reflected pulses, using seven different symbol durations \(T_p\). A variable delay of 770 ns, or 770 times the temporal width of the readout pulses, is readily observed (The right-most peak in Figure 21 is due to a parasitic reflection at frequency \(\nu_s\) that was not completely filtered out). Fine tuning of the variable delay on a sub-ns scale is shown in Figure 22. One should note again that the traces in both figures were acquired with averaging over multiple repetitions of delayed pulses.
Figure 21: Variable delay of reflected isolated and periodic readout pulses. The pulses were reflected from dynamic gratings, introduced by the 10th correlation peak of phase-modulated pump waves. The PRBS modulation clock rates $1/T$ were (left to right): 1.120 GHz, 1.108 GHz, 1.093 GHz, 1.078 GHz, 1.063 GHz, 1.048 GHz, 1.033 GHz. (The right-most peak, which is common to all PRBS rates, is due to a parasitic reflection.)

Figure 22: Variable delay of reflected isolated and periodic readout pulses. The pulses were reflected from dynamic gratings, introduced by the 10th correlation peak of phase-modulated pump waves. The PRBS modulation clock rate $1/T_p$ for the right-most peak was 1.118670 GHz. The clock rate was raised by 100 kHz, 200 kHz, 300 kHz and 400 kHz for the second through fifth peaks from the right, respectively.

3.4.2 Experimental setup #2 – real-time variable delay

Here, the experimental setup is similar to the one described in the previous section, with one important difference: the PRBS generator driving the common phase modulator was replaced with an arbitrary waveform generator (AWG). The AWG was
programed to repeatedly generate a perfect Golomb code of length $M_p = 63$, or alternatively a periodic PRBS of the same length. Both coding schemes were used and compared, in order to study their effect on residual side-lobe reflectivity. Throughout the experiment, the symbol duration was set to $T_p = 200 \text{ ps}$ ($N_o \approx 50$).

The optical power of pump 1 and pump 2 were $|A_1|^2 \sim 2 \text{ W}$ and $|A_2|^2 \sim 500 \text{ mW}$ respectively. The Brillouin frequency of the FUT in this experiment was $\Omega_a \sim 2\pi \times 10.86 \text{ GHz}$ and frequency difference between pumps and signal waves was $\Delta \nu \sim 48 \text{ GHz}$. The signal wave was amplitude modulated by return-to-zero (RZ) on-off-keying (OOK) PRBS patterns. The average power of the signals wave was amplified to 500 mW and the pattern was repeated at 10% duty cycle. Hence the peak power of the signal wave PRBS “on” pulses was 10 W. The FUT itself was stretched along a metal rod, in order to reduce temperature and strain variations (and consequently, reduce the spatial variance in both $\Omega_a$ and $\Delta \nu$).

The reflectivity of the 2-cm long DBG was very weak: only -64 dB. The reflected waveforms were therefore amplified to a peak power level of 1 mW by another EDFA and filtered again by a 4-GHz-wide optical bandpass filter to remove out-of-band amplified spontaneous emission (ASE). The reflected signal waves were detected by a photodetector, and the photo-currents passed through a 4-GHz-wide electrical low-pass filter and were sampled by an oscilloscope at a rate of 80 Gsamples/s.

The SNR of the detected signal waves can be estimated as follows: the noise equivalent power (NEP) of the detector used was 25 pW/\(\sqrt{\text{Hz}}\). At 4 GHz bandwidth, the thermal noise of the detector is therefore equivalent to 1.5 $\mu W$ of optical input power. The power of Amplified Spontaneous Emission (ASE) from the EDFA at the detection branch is:

$$P_{\text{ASE}} = F_n h \nu_r (G - 1) \Delta f,$$

where $F_n$ is the noise figure of the amplifier, $G$ is the power gain and $\Delta f$ is the optical filter bandwidth. With $F_n = 4$ and $G = 500$, the ASE power is 1.3 $\mu W$. Both
the detector noise and the noise due to beating between ASE and the reflected probe field are therefore considerably weaker than the mean photocurrent due to the amplified, reflected probe pattern itself. The power levels of the residual reflections of the pump and the input probe waves at the optical bandpass filter output were below $-25$ dBm. The OSNR of the reflected probe sequences is therefore expected to be limited by coding noise. For verification, the DBG in the probe path was temporarily replaced by a fixed equivalent attenuator of 64 dB and an output eye diagram was recorded (Figure 23). The quality of the output waveform was very good.

![Figure 23: Measured eye diagrams of 2.5 Gbit/s RZ, OOK PRBS probe waves, with the DBG temporarily replaced by an equivalent attenuator of 64 dB [58].](image)

The DBGs were first characterized using single, isolated probe pulses that were 200 ps long. Figure 24 shows the relative power reflected from the DBG as a function of time. The trace can be regarded as an “impulse response” of the localized DBGs. The average off-peak reflectivity of the Golomb-coded DBG is lower than that of the PRBS-coded one by a factor of 3.1. The corresponding ratio predicted by simulation is about 4.7 ($\sigma^2_R = 5.3$ and 25 for the Golomb code and PRBS, respectively). Note that no averaging was performed in these traces.

The impulse response of Golomb-coded DBGs for several slightly different writing sequence clock rates is shown in Figure 25. As described in the previous section, a high-order correlation peak could be scanned along the FUT by slightly tuning the rate of the coding sequence. Clock rate variations in the range of 4.81–5.25 GHz effectively moved the DBG along the entire 1 m-long FUT, introducing delay variations of reflected probe pulses by up to 10 ns.
Figure 24: Relative reflected power from stationary and localized DBGs as a function of time (“impulse response”). The interrogating probe waveform was a single, isolated pulse of 200 ps duration. Blue: Golomb-coded DBG; red: PRBS-coded DBG [58].

Figure 25: Variable delay of single, isolated probe pulses using Golomb-coded stationary and localized DBGs [58].

Figure 26 shows the eye diagrams of delayed probe waves, modulated by RZ OOK PRBS at 1 Gbit/s. The waveforms were reflected from Golomb-coded (top) and PRBS-coded (bottom) DBGs. An open eye is obtained using Golomb coding, whereas the eye diagram for the PRBS coded DBG is closed. The corresponding simulated eye diagrams are shown in Figure 27 [58]. Good agreement between experiment and simulations is evident.

The results indicate that the coding noise of the Golomb-encoded DBG is lower than that of the PRBS-inscribed one. Figure 28 and Figure 29 show the experimental and simulated eye diagrams of reflected RZ OOK PRBS probe data at 2.5 Gbit/s. The
Golomb- and PRBS-coded DBGs were the same as those in Figure 26 and Figure 27. The eye diagram obtained using the Golomb-coded DBG is only marginally open at this data rate.

Figure 26: Measured eye diagrams of 1 Gbit/s RZ OOK PRBS probe waves, reflected from DBGs that were written by Golomb code modulation of the pump waves (top) and PRBS modulation (bottom). The pump modulation rate was 5 Gbit/s [58].

Figure 27: Simulated eye diagrams of 1 Gbit/s RZ OOK PRBS probe waves, reflected from DBGs that were written by Golomb code modulation of the pump waves (top) and PRBS modulation (bottom). The pump modulation rate was 5 Gbit/s [58].
Figure 28: Measured eye diagrams of 2.5 Gbit/s RZ OOK PRBS probe waves, reflected from DBGs that were written by Golomb code modulation of the pump waves (top) and PRBS modulation (bottom). The pump modulation rate was 5 Gbit/s [58].

Figure 29: Simulated eye diagrams of 2.5 Gbit/s RZ OOK PRBS probe waves, reflected from DBGs that were written by Golomb code modulation of the pump waves (top) and PRBS modulation (bottom). The pump modulation rate was 5 Gbit/s [58].

3.5 Conclusions

In this chapter, phase-coded SBS was introduced as a technique for the generation of localized and stationary acoustic gratings. The effective generation of
the dynamic acoustic gratings is confined to discrete correlation peaks, the length of which is governed by phase modulation symbol duration rather than the much longer acoustic lifetime. The separation between neighboring peaks, and hence the unambiguous measurement range, can be made arbitrarily long and the free range can routinely exceed $10^9$ grating lengths using basic PRBS generation equipment.

When implemented in PM fibers, unambiguous characterization of the grating properties is allowed since the activating and readout signals can be entirely distinct waves, in both frequency and polarization. In addition, dynamic gratings over PM fibers offer several advantages: the use of a third and fourth optical frequencies improves the measurement SNR; the sensitivity of the PM fiber birefringence to both temperature and strain variations adds another dimension to sensing applications; and the need for polarization tracking or scrambling is circumvented. Reflections from multiple short and stationary DBGs, induced by phase modulation of the writing pumps, were successfully used in microwave-photonic multi-tap filters [50].

The availability of localized and time-independent gratings is used in the all-optical variable delay of reflected readout data. In principle, the gratings provide a 'movable mirror' that can be scanned over paths that are many μs-long, and accommodate signals of several GHz bandwidth. In addition, the wavelength conversion that accompanies the delay is minimal (an offset by the Brillouin shift).

Unfortunately, both analysis and simulations show that the accumulative effect of scattering noise from residual gratings spanning the entire length of the fiber is severely restricting the reflection OSNR.

Analysis and simulations predict that a reasonable OSNR in reflection from PRBS coded DBGs cannot be maintained, unless delay variations are restricted to less than 1 ns. The unique correlation properties of perfect Golomb codes are a promising way of reducing off-peak reflectivity. Simulations suggest that a delay-bandwidth product of 45 with an output OSNR of 13 dB can be achieved by Golomb coding.

The delay of 1 Gb/s OOK signals by as much as 10 ns was demonstrated experimentally, with an OSNR that was sufficient to retain an open eye diagram. Golomb coding showed clear advantage over PRBS coding, although the difference
was somewhat smaller than was anticipated by the analytical model. The difference might suggest that the favorable correlation properties of Golomb codes are more sensitive to imperfect modulation caused by instrument limitations than PRBS codes. Nevertheless, the experiments validate the added value of Golomb coding, and provide a successful demonstration of a variable delay with delay-bandwidth product of 10, an order of magnitude larger than those of broadband SBS slow light-based setups [56].
4 Phase-coded SBS sensing

4.1 Introduction

Having presented the phase-coded SBS technique in Chapter 2 and demonstrated its capability to confine the SBS interaction to an arbitrary discrete location in Chapter 3, we now turn to the main focus of this work – distributed sensing. In this chapter, we present a scheme for distributed sensing based on phase-coded SBS interactions. The concept is experimentally validated, and high resolution sensing with comparatively long unambiguous measurement range is demonstrated.

Initial experiments showed that although the presence of multiple, spatially-periodic correlation peaks is no longer a limiting factor, time constraints and accumulated coding noise restrict the number of resolution points that can actually be scanned within a reasonable experimental duration. An improved approach to reduce the acquisition time and increase the sensing range is proposed and demonstrated. The technique relies on the probing of multiple correlation peaks by a single pump pulse, thus reducing coding noise and improving temporal efficiency. The overall performance is further improved by using a sequence of pump pulses, instead of a single pulse. This ‘dual hierarchy’ coding of both phase and amplitude is ultimately demonstrated, in an experiment in which an 8.8 km long fiber is fully covered, with spatial resolution of 2 cm. All 440,000 resolution points are properly addressed, and a few cm-wide hot-spot located towards the far end of the fiber is properly identified. The experimental uncertainty in the estimate of the local Brillouin frequency shift in this long-range experiment is 3.5 MHz.

4.2 Sensing with a constant-amplitude pump wave

4.2.1 Principle of operation

A conceptual sketch of a phase-coded SBS sensing system is depicted in Figure 30. Both pump and signal waves are drawn from the output of a single monochromatic laser. The signal wave is offset in frequency by \( \Omega \), which is on the order of the Brillouin frequency shift in the fiber under test. Both waves are co-modulated by a common PRBS phase code, and launched from opposite ends of the fiber under test. Careful
timing of the two phase modulators provides a spatial scan of the code correlation peak position along the fiber, and allows for a random access to arbitrary points of interest. The Brillouin gain spectrum in each point is reconstructed through scanning Ω. Perfect Golomb codes were not used in this experiment (for discussion, see Section 4.2.3).

4.2.2 Experimental results

A detailed drawing of the specific setup in our experiments is given in Figure 31. Light from a distributed feedback (DFB) laser diode of frequency ωₜ was used as the source of both pump and signal. The DFB output passed through an electro-optic phase modulator, driven by a PRBS generator. The generator clock rate 1/T was in the range of 8–12 GHz, corresponding to Δz between 1.3 cm and 0.9 cm, and the code length was M_p = 2^{15} – 1. The modulator driving voltage was adjusted to match its V_c. The modulated wave was split into pump and signal paths. In the signal branch, an amplitude electro-optic modulator was used to generate two modulation sidebands at frequencies ωₜ ± Ω, the lower of which was subject to SBS amplification. The modulator was biased to suppress the carrier wave at ωₜ. The signal wave was amplified and launched into one end of a fiber under test. Light in the pump branch was amplitude-modulated by a low-frequency sine wave, amplified to 500 mW, and launched into the opposite end of the FUT. A polarization scrambler was used in the

Figure 30: A schematic illustration of a phase-coded SBS sensing setup [59].
pump arm, in order to avoid potential polarization-related fading of the SBS interaction [60].

Arbitrary locations along the FUT were addressed as described in section 3.4.1: the position of the peak could be scanned through slight changes to the PRBS symbol duration, where a 1 MHz variation in the clock rate corresponded to an offset of the correlation peak by 34 cm. At the same time, the code was sufficiently long to guarantee that only one peak was generated within the FUT itself.

The output signal was filtered by a FBG which retained only the sideband of interest at $\omega_s$, and detected by a photo-diode. The detected current was analyzed by a lock-in-amplifier, tuned to the low modulation frequency of the pump wave. Hence potential contribution from spontaneous Brillouin scattering at the signal wave frequency was suppressed. The Brillouin gain spectrum in each fiber locale was reconstructed through measuring the power of the amplified output signal wave as a function of $\Omega$.

**Figure 31:** Experimental setup for phase coded SBS based distributed sensing with a constant-amplitude pump wave. LF: low frequency.
Figure 32 shows measurements along a single-mode fiber, which consisted of several non-identical segments spliced together. One section was 1 cm short. In addition, a 7 cm-long section of the fiber was locally heated by a high-current resistor. Spatial variations in the Brillouin gain spectrum are evident in the heated and dissimilar sections. Figure 33 shows the analysis of a 40 m-long fiber, in which a 1 cm-long heated section is resolved. The experimental uncertainty in the estimate of the local Brillouin frequency shift is 0.5 MHz.

Figure 32: Brillouin gain mapping of a fiber under test, comprised of a 1 cm-long section of spliced dissimilar fiber (point B), and a 7 cm-long heated section (point D). The Brillouin gain spectra at different points along the fiber are resolved using the random access sensing method (see insets for addressing points A through E). The Brillouin frequency shifts for points B and D are different from the background values (points A, C, E). The lower curve shows the extracted Brillouin frequency shift after addressing sequentially all positions. Note the sharp edges of the dissimilar spliced fiber, as opposed to the gradual profile of the heated section (heat conduction) [59].
4.2.3 Discussion

The results obtained with the continuous pump approach represent a breakthrough in B-OCDA performance, in the sense that the range of unambiguous measurements is effectively decoupled from spatial resolution. A truly random access to each point along a fiber under test is allowed.

However, a significant drawback of this scheme is that each resolution point must be accessed separately, in contrast to B-OTDA systems which address the entire fiber with a single scan. The minimal time it would take to map a fiber of length $L$, disregarding latencies associated with the switching of laboratory instruments, would be:

$$T_{map} = T_D \times N_f \times N_{avg} \times L / \Delta z ,$$

where $N_f$ is the number frequency offsets sampled, and $N_{avg}$ is the number of repetitions that are averaged for the purpose of noise reduction. For example, taking
$L = 2 \text{ km (} T_p = 20 \mu s \text{)}, \Delta z = 2 \text{ cm and } N_v = 30$, the net time-of-flight required for a single repetition is:

$$\frac{T_{\text{map}}}{N_{\text{avg}}} = 60 \text{ s}.$$  \hfill (4.2)

Long distance measurements are also impaired by accumulated coding noise. Since the magnitudes of both pump and signal waves are continuous, undesired, off-peak acoustic fields exist at every moment in time throughout the entire fiber under test. The noise term $G_N$ is defined as pump reflections from off-peak acoustic gratings, whereas the signal term $G_S$ is determined by the reflections from the main peak. The expectation value of the OSNR in measurement of the probe gain is given by Eq. (2.8):

$$\text{OSNR} = \frac{G_S}{G_N} = \frac{N_0^2}{N_{\text{bins}} \sigma_R^2}.$$  \hfill (4.3)

Long measurement ranges require long sequences ($M_p > L / \Delta z$) and Golomb codes give no advantage over PRBS codes in terms of reducing $\sigma_R$ for $M_p \gg N_0$ (see Figure 13). Therefore, Eq. (4.3) may be reduced to the special case of PRBS coding:

$$\text{OSNR} = \frac{G_S}{G_N} = \frac{2 \tau v_g}{L} \approx \frac{2}{L},$$  \hfill (4.4)

where the length $L$ is given in meters. It is important to note that while $G_S$ is constant in time, $G_N$ is not. At each moment in time $t$, noise stems from the accumulated reflections from the acoustic field in all off-peak 'bins' along the fiber. However, each bin reflects a different bit in the pump sequence. Since the bins are considered to be uncorrelated, then at time $t + T_p$ the reflections of the pump become uncorrelated to those of time $t$.

Given the above, the OSNR may be improved by integrating the input signal over a period of time $T_\text{int}$ [61]:

70
\[
\text{OSNR} \approx \frac{2}{L} \sqrt{\frac{\tau_{\text{int}}}{T_p}}.
\] (4.5)

Since each scan takes a minimum of \(T_D\), we may take \(\tau_{\text{int}} = T_D\), without increasing \(T_{\text{map}}\) by more than a factor of 2. In this case, returning to the example given above, the mean OSNR in a single scan of a 2 km-long fiber is \(\sim 0.31\). The SNR can be improved by a factor of \(\sqrt{N_{\text{avg}}}\), with averaging over \(N_{\text{avg}}\) repetitions of each measurement. An SNR of 3 dB would therefore require approximately 45 averages, which take up about 90 minutes of net-time-of-flight in data acquisition.

It should be noted again that this estimate does not take into consideration the latencies that are associated with the switching of standard, multi-purpose laboratory equipment a very large number of times. Such switching is necessary in the scanning of the PRBS clock rate and of the frequency offset \(\Omega\). The above example, the analysis of 2 km of fiber with 2 cm resolution, would require an unthinkable amount of time in a standard research laboratory. The above considerations also assume that coding noise constitutes the dominant noise mechanism, and that other sources such as additive detector noise, shot noise, intensity variations of the light source, and ASE may be disregarded. As seen in previous chapters, this assumption is justified in many phase-coded B-OCDA experiments.

### 4.3 Pulsed pump

#### 4.3.1 Principle of operation

In this scheme, the complex envelopes of the pump and signal, at their respective points of entry into the fiber, are co-modulated by a high-rate phase sequence in a similar manner to the previous approach. However, while the magnitude of the signal wave remains continuous, an additional amplitude modulation by a single, comparatively long pulse is overlaid on top of the phase sequence modulation of the pump wave:

\[
A_p(z = 0, t) = A_{p0} \text{rect} \left( \frac{t}{T_o} \right) \sum_n c_n \text{rect} \left[ \frac{t - nT_p}{T_p} \right] = A_p(t)
\] (4.6)
In Eq. (4.6) $A_{p0}$ is a constant magnitude and $T_a$ is the duration of the pump amplitude pulse. The phase sequence symbol duration $T_p$ and the pulse duration $T_s$ are chosen so that $T_a \approx M_p T_p > \tau$. The simulated acoustic field $|\rho(z, t)|$ over a 6 m-long fiber section, subject to the modulation scheme of pump and signal described above, is shown in Figure 34. A uniform $\Omega_a$ was assumed, and the frequency offset $\Omega$ was chosen to match that value. A perfect Golomb code ($M_p = 127$, $T_p = 200$ ps, see Appendix) was used in the phase modulation of both pump and signal, and a 26 ns-long amplitude pulse was superimposed on the phase-modulated pump wave.

Figure 34: Simulated magnitude of the acoustic wave density fluctuations (in normalized units), as a function of position and time along 6 m of fiber. Both pump and signal waves are co-modulated by a perfect Golomb phase code that is 127 bits long, with symbol duration of 200 ps. The pump wave is further modulated by a single amplitude pulse of 26 ns duration. The acoustic field, and hence the SBS interaction between pump and signal, is confined to discrete and periodic narrow correlation peaks [7].

As expected in phase-coded B-OCDA, the acoustic field is confined to a discrete set of spatially-periodic correlation peaks, whose width $\Delta z$ equals 2 cm in this case. The separation between neighboring peaks is $M_p \Delta z$. Unlike the previous scheme, however, the temporal duration of each correlation peak is restricted to the order of $T_s$, and the peaks do not overlap in the time domain. The power of the output signal,
at any given instance, would be affected by SBS amplification in a single correlation peak or by none at all.

Figure 35 shows the simulated output signal power as a function of time $|A_s(z=0,t)|^2$. The trace consists of a series of amplification events, each of which can be unambiguously related to the SBS interaction at a specific correlation peak of a known location. A single trace therefore provides information on $L/(M_p \Delta z) \gg 1$ fiber positions.

The locations of the correlation peaks can be offset in $\Delta z$ increments with proper retiming of the phase modulation of the pump and signal (see section 3.4.1). The correlation peaks would return to their initial locations every $M_p$ steps, hence $M_p$ scans would be sufficient for the mapping of the Brillouin gain over the entire fiber, for each choice of $\Omega$. This number of scans does not depend on the fiber length, and is orders-of-magnitude smaller than the number of resolution points $L/\Delta z$. The combined technique retains the high resolution and long range of unambiguous
measurement that is provided by phase-encoded B-OCDA, with an acquisition time that is potentially much reduced.

4.3.2 Experimental results

Figure 36 shows the experimental setup for high-resolution, extended-range Brillouin analysis using the pulsed pump technique. Both pump and signal waves were drawn from a single laser diode source at 1550 nm wavelength. An electro-optic phase modulator at the laser output was driven by an arbitrary waveform generator (AWG). The AWG was programmed to repeatedly generate a 127 bits-long perfect Golomb code (see Appendix), with a symbol duration on the order of 200 ps. The output voltage of the generator was adjusted to match $V_\pi$ of the modulator.

The phase-modulated light was split into pump and signal branches by a 50/50 fiber coupler. Light in the pump branch was amplitude-modulated by a sine-wave of frequency $\Omega$, in suppressed-carrier format. The upper modulation sideband was retained by a narrow-band FBG, whereas any residual carrier and the lower sideband were blocked. The pump wave was then modulated again by a second amplitude electro-optic modulator, driven by repeating pulses with a low duty cycle. The pulses duration of 26 ns was chosen to approximately match the period of the Golomb code, and their period of 20 $\mu$s exceeded the time-of-flight through the fibers under test. Pulses were then amplified by an EDFA to an average power of 200 mW. A polarization scrambler was used to prevent polarization-related fading of the SBS interaction [60]. Light in the signal branch was delayed in a 25 km-long fiber imbalance.

The signal and pump waves were launched into opposite ends of a 1600 m long FUT. The signal wave at the output of the FUT was detected by a photo-detector of 200 MHz bandwidth, and sampled by a real-time digitizing oscilloscope. Each trace was averaged over 512 pump pulses, in order to overcome polarization scrambling variations, coding and thermal noises. A few cm-long hot-spot was introduced to the FUT, towards the output end of the pump wave.
Figure 36: Experimental setup for combined B-OTDA and B-OCDA distributed sensing [62].

Figure 37 shows examples of the output signal power trace as a function of time, with $\Omega$ adjusted to the Brillouin shift of the fiber under test at room temperature (top panel), and at the temperature of the hot spot (bottom panel). Only small segments of the traces, representing the output end of the pump wave, are shown for clarity. Each peak corresponds to SBS amplification at a single correlation peak. The final peak is in overlap with the hot spot. The peak does not show at the top panel, but reappears in the bottom panel.

Figure 37: Measured output signal power as a function of time, taken with $\Omega$ adjusted to the Brillouin shift of the fiber under test at room temperature (top panel), and at the temperature of the hot spot (bottom panel). Only short, magnified segments of each trace are shown, corresponding to the output end of the pump wave [62].
Figure 38 shows the SBS signal power gain as a function of $\Omega$ and $z$. The experimental procedure effectively reconstructed the Brillouin gain spectra at all 80,000 resolution points using only 127 scans for each choice of $\Omega$. The recovered values of $\Omega_b(z)$ are shown in Figure 39. The hot spot is well recognized (see inset).

Figure 38: Measured Brillouin gain map as a function of frequency offset between pump and signal, and of position along a 1600 m-long fiber under test. A hot spot was located towards the output end of the pump wave. The map was reconstructed using only 127 scans per frequency offset, according to the pulsed-pump method. The complete map is shown on the left, and a zoom-in on the hot spot region is shown on the right [62].

Figure 39: Brillouin frequency shift as a function of position, as extracted from the experimental Brillouin gain map of Figure 38 above [62].

4.3.3 Discussion

In the pulsed pump, phase-coded SBS sensing technique described in this section, multiple gain events can be resolved within a single scan. This capability is in
contrast to the previous, continuous-amplitude pump technique, where only one a single correlation peak was addressed in each trace. This advantage, of course, translates into significantly reduced acquisition times. The minimal time for data acquisition (again, not including instruments-related constraints) is:

\[ T_{\text{map}} = T_D \times N_v \times N_{\text{avg}} \times M_p. \]  

(4.7)

Compared with Eq. (4.1), the time for acquisition of a complete map is reduced by a large factor of \( L / (M_p \Delta z) \). Furthermore, the number of required scans does not depend on the length of the fiber but on the length of the phase sequence, \( M_p \). Note, however, that \( M_p \) cannot be made arbitrarily short, or else a pump pulse of duration \( T_a \) might be in simultaneous overlap with more than one correlation peak. The duration \( T_a \), in turn, must be kept long enough to allow the acoustic field to reach a steady state, hence:

\[ M_p > \frac{T_a}{T_p} > \frac{\tau}{T_p}. \]  

(4.8)

Taking \( M_p = 127 \), \( N_v = 30 \) and \( L = 2 \) km, the time required for a complete scan of the FUT without repetitions is:

\[ \frac{T_{\text{map}}}{N_{\text{avg}}} = 76 \text{ ms}. \]  

(4.9)

This duration is significantly shorter than the 1 minute required to obtain the same data in the continuous pump method.

The second main advantage of time-gating the pump wave is that the off-peak acoustic waves are restricted to the spatial extent of the pump, rather than span the entire FUT. Using Eq. (4.3), where \( N_{\text{bins}} \) is now the number of bins in overlap with the pulse, the mean SNR as a result of coding noise alone at the optimal point of \( T_a = M_p T_p \) is:
\[
\text{OSNR} = \frac{G_s}{G_N} = \frac{N_0^2}{M_p \sigma_R^2}
\] (4.10)

We take for example a 127 long Golomb code at \( T_p = 200 \text{ ps} \). In that case, \( \sigma_\rho^2 \approx 10.6 \), \( N_0^2 \approx 2500 \), and:

\[
\text{OSNR} \approx 1.85
\] (4.11)

Comparison with the previous section reveals that not only does the time-gating scheme lead to a much faster data acquisition, but it also does so with a much higher OSNR.

The signal gain is evaluated over a duration \( T_{\text{int}} \) that is about 5 ns long. This interval is chosen near the peak of each amplification event (see Figure 35). It cannot be further increased, since the acoustic field does not maintain its peak value for longer durations. Given this integration window:

\[
\text{SNR} \approx \frac{N_0^2}{M_p \sigma_R^2} \left( \frac{T_{\text{int}}}{T_p} \right) \approx 9.2
\] (4.12)

From Eq. (4.12) we learn that, at least in principle, no averaging is required to achieve a sufficient suppression of coding noise. However, with coding noise reduced, the measurement OSNR might become restricted by other mechanisms. These include thermal and shot noise, ASE, polarization scrambling variations and phase-to-amplitude modulation of the high-rate code taking place at the output FBG filter [63]. The suppression of these noise mechanisms does require averaging. In the experiment described here, 512 repetitions were required to achieve the desired OSNR, representing as net time-of-flight that is under 40 seconds. Even with averaging, the gated pump approach is able to reduce the required time-of-flight in data acquisition by two orders of magnitude. This reduction makes the analysis of few-km-long fiber with cm-scale resolution feasible, as demonstrated above.

Some noise sources, such as ASE, thermal noise and shot noise, can be overcome at least in part by amplifying the pump power. However, pump power is limited by the onset of Kerr nonlinearity, a restriction that becomes more severe with
the length of the FUT. Complete analysis of competing nonlinear mechanisms and their effect on the SBS pump wave is beyond the scope of this work. However, an estimate of the maximum allowed pump power may be obtained. For standard single-mode fibers, a typical value for the Kerr nonlinearity coefficient is \( \gamma = 1.3 \text{ rad/(km-W)} \). The non-linear phase accumulated by the pump due to self-phase modulation (SPM) is:

\[
\phi_{\text{non-linear}} = \gamma |A_{00}|^2 L
\]  

Effects related to Kerr nonlinearity become appreciable when \( \phi_{\text{non-linear}} \) reaches the order of \( \pi \) radians. In the experiment described above, the peak power of the pump was \(~1 \text{ W} \). For \( L = 1600 \text{ m} \), \( \phi_{\text{non-linear}} \) indeed approaches \( \pi \) radians. Therefore, any further increase in the pump power level, or an extension of range at the same power, might give rise to intolerable nonlinear distortion of the phase code, spectral broadening of the pump spectrum, reduction of SBS gain and larger measurement errors.

Lastly, time-gating schemes over B-OCDA were also considered by Hotate and co-workers. Dynamic strain measurements of a 200 m long fiber with 8 cm resolution at a rate of 1000 point/s were recently demonstrated [64]. Denisov and co-workers have optimized a similar scheme, and were able to extend the applicable sensing range up to 17.5 km, with 8 mm spatial resolution [65]. The setup provides, in principle, more than 2 million potential resolution points. However, only a small subset of these points could be actually addressed in a given experiment.

4.4 Amplitude-coded pump

4.4.1 Principal of operation

Here, a constant-magnitude signal is once again modulated by a high-rate phase sequence with bit duration \( T_p \). The phase of the pump wave is also modulated by the same sequence, as before. This time, however, the amplitude of the pump wave is also modulated by a second binary sequence \( d_m \) of length \( M_s \) and symbol duration \( T_s \).
\[ A_p(z=0,t) = A_{p0} \sum_{m=1}^{M_p} d_m \text{rect} \left( \frac{t-mT_p}{T_o} \right) \times \sum_n c_n \text{rect} \left( \frac{t-nT_p}{T_p} \right) = A_p(t) \quad (4.14) \]

The symbol durations of the phase and amplitude sequences are chosen so that:

\[ \frac{1}{2} M_p T_p > T_o > \tau \quad (4.15) \]

Figure 40(a) shows the calculated magnitude of the acoustic field \( |\rho(z,t)| \) subject to the boundary conditions in Eq. (4.14). A magnified view of the center of the fiber is shown in Figure 40(b). The calculation parameters were \( T_p = 200 \text{ ps} \), \( M_p = 499 \), and \( T_o = 40 \text{ ns} \), and the length of the amplitude modulation sequence \( d_m \) was 26 bits. The length of the fiber was 20 m. Once again, the acoustic field is spatially confined to the discrete and narrow correlation peaks of the high-rate phase modulation sequence. In contrast to the single-pulse pump technique, however, the acoustic field at each correlation peak is switched on and off by the amplitude modulation of the pump wave, on a time scale of \( T_o \).

---

**Figure 40:** Simulated magnitude of the acoustic wave density fluctuations (in normalized units), as a function of position and time along a 20 m-long fiber section. Both pump and signal waves are co-modulated by a repeating perfect Golomb phase code that is 499 bits long, with symbol duration of 200 ps. The pump wave is further modulated by a 26 bits-long amplitude sequence, with a pulse duration of 40 ns. The acoustic field at the three correlation peaks of the phase codes is turned on and off by the overlaying amplitude modulation of
The SBS interaction at each correlation peak contributes a series of amplification events to the output signal wave, which replicates the amplitude modulation pattern of the pump wave itself. These replicas of the pump amplitude sequence are extended in time and are in large overlap, and therefore they may not be separated directly. However, the contributions of individual correlation peaks to the overall trace may be resolved in post-processing of the raw output signal data, via cross-correlation with a reference code. This mechanism, known as incoherent sequence compression, is schematically illustrated in Figure 41. The synthesis of amplitude sequences and their corresponding reference codes is addressed elsewhere [67-69]. The correlation properties of the amplitude code are critical to the successful separation between SBS interactions from multiple correlation peaks.
The compressed form of the probe signal, as illustrated in Figure 41, is analogous to that obtained by the previous, single-pulse hybrid B-OTDA / B-OCDA scheme. However, the magnitude of each compressed peak corresponds to the sum of $M_a$ amplification pulses added together. The OSNR of the processed traces may therefore be considerably better than that obtained by a single pump pulse. The improvement in OSNR scales with the square root of the length of the amplitude sequence. In this work we used a 9785 bit-long sequence [70], which may improve the OSNR by a significant factor. The hierarchal duel-layer encoding of both phase and
amplitude could reduce the acquisition time that is necessary for the construction of Brillouin gain spectra with sufficient fidelity.

4.4.2 Experimental results

The experimental setup is shown schematically in Figure 42. Light from a laser diode source at 1550 nm wavelength passed through an electro-optic (EO) phase modulator, driven by the output voltage of an AWG. The AWG was programmed to repeatedly generate a 211 bits-long Golomb code with $T_p$ of 200 ps ($\Delta z$ of 2 cm). The output voltage of the AWG was adjusted to match $V_\pi$ of the EO modulator.

The modulated signal was split into two branches. Light at the pump branch passed through a single-sideband EO modulator, driven by a RF sine wave of frequency $\Omega \approx \Omega_\omega$. The pump was then amplitude-modulated by an aperiodic 9,785 bits-long binary sequence using another AWG, with $T_a$ of 20 ns. Lastly, the pump wave was amplified to an average power level of 150 mW by an EDFA and launched into an 8.8 km-long FUT. At the signal branch, a 60 km-long fiber delay was used to allow for the scanning of the correlation peaks positions. A polarization scrambler was used to avoid polarization-induced fading of the SBS interactions. The signal was then launched into the opposite end of the FUT. The output signal was detected by a 200 MHz-wide photodetector, and the detector output was sampled by a digitizing oscilloscope and averaged over $N_{avg} = 256$ repetitions. Each averaged trace was cross-correlated with a carefully constructed reference sequence, using offline digital processing [70]. As many as 2,100 correlation peaks were simultaneously addressed in each trace.
Figure 42: Experimental setup for long-reach, high-resolution distributed Brillouin fiber analysis. SSB: single sideband. Solid lines denote fiber paths, and dashed lines represent radio-frequency electrical cables [70].

Figure 43(top) shows a map of the Brillouin gain as a function of $\Omega$ and $z$ [71]. The analysis of all 440,000 resolution points is presented. Figure 43(bottom) shows a magnified view of the gain map in the vicinity of a 7 cm-long hot-spot, located towards the output end of the pump wave. The hot-spot is properly recognized in the measurements. The setup is capable of resolving even shorter events, on the order of $\Delta z$, however this limit was not attempted in the experiment. The experimental uncertainty $\sigma_v$ in $\Omega_\phi$, estimated by the standard deviation of the local difference $[\Omega_\phi(z + \Delta z) - \Omega_\phi(z)]/2\pi$, was ±3.5 MHz.
Figure 43: Top - Measured Brillouin gain of the output signal wave (in arbitrary units), as a function of correlation peak position, and of the frequency difference between pump and signal waves. 440,000 resolution points are addressed. Bottom - Magnified view of the gain map in the vicinity of a 7 cm-long hot spot, located towards the output end of the pump wave [71].
4.4.3 Discussion

In the scheme presented in this section, the amplitude of the pump wave is sequence-coded, in addition to the common phase modulation of both pump and probe. The phase code confines the SBS interaction to discrete correlation peaks, as before. A large number of peaks contribute extended and overlapping amplification events, which may be resolved by correlation analysis of the output probe wave trace. The careful construction of the amplitude modulation sequences results in little to no cross-interference among over 2,000 individual events (see [70] for details). Distributed sensing of over 8.8 km of fiber with a 2 cm resolution was successfully demonstrated. The entire set of 440,000 potential resolution points was addressed. The number of points is five times larger than that of the setup based on a single-pulse pump.

We again consider the time required to fully address all the resolution points, using the same example of a 2 km long FUT. In this case:

\[ T_{\text{map}} = (T_D + M_a T_a) \times N_v \times M_p \times N_{\text{avg}}. \]  \hspace{1cm} (4.16)

With \( M_p = 211 \) and \( M_a T_a \approx 200 \mu s \), and without repetitions, the net time-of-flight would be:

\[ \frac{T_{\text{map}}}{N_{\text{avg}}} = 1.4 \text{ s} \]  \hspace{1cm} (4.17)

Compared with (4.9), the required time for a single repetition is now greatly increased, mainly due to the length of the amplitude sequence. The extended amplitude sequence appears to be a drawback. However, the magnitude of the SBS gain events following the post-processing of a single trace is equivalent, at least in principle, to \( M_a \) repetitions of the single-pulse measurements taken using the previous setup. Therefore, many noise mechanisms such as shot noise, additive detector thermal noise, ASE etc. are effectively suppressed by a factor of \( \sqrt{M_a} \). Similar suppression could only be achieved with the averaging over \( M_a = 9785 \) repetitions of the single-pulse traces proposed in the previous scheme. Therefore, the dual-
hierarchy coding of both phase and amplitude has great potential for reducing the acquisition duration, rather than increasing it.

Taking full advantage of this potential for much faster acquisition times is not straightforward. From the standpoint of technical aspects, variations due to polarization scrambling in our experiment could not be effectively suppressed by the cross-correlation process. This is due to the fact that the coherence time of the scrambling variations was relatively long: $T_{\text{scrambler}} \approx 1 \mu s$. Suppression of this noise required additional repetitions [70]. However, polarization scrambling may be avoided altogether using recent and more advanced polarization switching and diversity schemes [72].

On a more fundamental note, unlike the single-pulse experiment, off-peak acoustic fields are once again introduced over the entire extent of the fiber under test and at all times. In this regard, the amplitude coding represents a step back towards the coding noise issues that restricted the initial, constant-magnitude scheme. The powerful incoherent compression protocols provide strong mitigation of this coding noise, however its residual extent is very difficult to quantify analytically. The OSNR subject to the boundary condition of dual-hierarchy coding remains the subject of extensive numerical analysis in our group. While this research is ongoing, it is already clear that the dual-layer coding allows for the monitoring of longer fibers and a larger number of high-resolution points which may not be reached using the previous, single-pulse approach. On the other hand, it is also established that the performance of this more advanced scheme is restricted by residual coding noise, and that the highly optimistic above estimates of overall noise reduction cannot be reached. Lastly, this scheme is associated with considerable complexity, especially in the post-processing of large amounts of data.

### 4.5 Application example – composite material monitoring

Composite materials consist of strength members such as glass or carbon fibers, embedded in a polymer matrix. Composites provide favorable ratios of strength to weight, corrosion resistance, and freedom in forming complex three-dimensional shapes. Many critical parts in the aerospace, energy, transportation and medical
sectors are made of composite materials. The consequences of failure in such parts could be catastrophic, a situation that warrants reliable structural monitoring. In most cases, static and dynamic structural analysis of composite material parts relies on point sensors, such as strain gauges, accelerometers or FBGs. In a series of works by Tur and collaborators, FBGs were used in the monitoring of curing processes of composite patch repairs [73] and in measurement of Lamb waves [74]. Distributed Brillouin sensing was introduced to monitoring in aircrafts in 2001 [Y30], and the analysis of full-scale composite structures with 1 m resolution was reported in 2003 [Y31]. Minardo et al. recently used dynamic Brillouin sensing in vibration modal analysis of a composite structure, with a spatial resolution of 20 cm [75].

Here, we employ high-resolution Brillouin analysis in the monitoring of beams made of composite materials, which are being routinely used in aerospace and wind turbine applications. Standard single-mode sensing fibers were embedded during fabrication of the beams. Distributed strain measurements were performed based on the pulsed-pump phase-coded SBS configuration, as described in Section 4.3. Here, a 63 bits-long Golomb code with bit duration of 400-800 ps was used. This duration corresponds to a spatial resolution of 4-8 cm. The pump pulses were 25-ns long with peak power of $\sim 2$ W. The frequency offset $\Omega$ was scanned in intervals of 2 MHz, and each output trace was averaged over 512 repetitions. The composite beams were constructed of structural layers of glass fibers sheets, bound by epoxy resin.

In a first experiment, high-resolution B-OCDA was continuously performed for over 30 hours during and following production of a 1 m-long beam. Measurements were taken every 20 minutes, with a spatial resolution of 4 cm. The uncertainty in the measured $\Omega_g$ was $\pm 1$ MHz. The values of $\Omega_g$ as a function of position and time are shown in Figure 44. The edges of the beam are clearly identified in the results: the values of $\Omega_g$ in fiber segments within the beam vary with time, whereas those in the fiber leads outside the beam remain nearly constant. The results indicate an initial heating of the beam by up to 20-25 °C over several hours, during the exothermic curing process of the epoxy resin. The beam reaches thermal equilibrium with the room
environment after about 20 hours. A residual compressive strain of about 200 με remains in the beam.

Figure 44: Measured changes in the Brillouin frequency shift (in MHz), as a function of position within a 1 m-long composite beam, monitored continuously over 30 hours following its production. Measurements were performed with 4 cm resolution and 1 MHz accuracy, every 20 minutes.

In a second experiment, a 2 m-long beam with embedded fibers was produced for subsequent testing in a three-point bending setup. A picture of the setup is provided in Figure 45 (left side); it shows the beam simply supported by two cylindrical rollers, bridging a gap of length \( L = 1800 \) mm. A sketch is also provided in this figure (bottom right) wherein \( 0 \leq x \leq L \) denotes a position variable, and \( w(x) \) denotes the beam deflection at any position due to a force with magnitude \( P \) applied at the center span \( (x = L/2) \). Mechanical indicators were placed underneath the beam, providing an independent measurement of deflections at six discrete \( x \) values. The beam had a rectangular cross-section, with \( B = 120 \) mm (width) and \( H = 20 \) mm (height); the FUT in this specific experiment was located underneath the outermost structural layer, along the entire length of the beam, and at a vertical offset distance from the centroid of \( z = 9.5 \) mm (see top right side of Figure 45).
According to the Euler-Bernoulli theory [76], the beam deflection for the specific setup is given by:

\[
w(x) = \frac{P}{48EI} \left[ 3L^2x - 4x^3 + 8 \left( x - \frac{L}{2} \right)^3 + 8 \left| x - \frac{L}{2} \right| \right],
\]

in which \( E \) is the material’s Young’s modulus and \( I = \left( BH^3 \right)/12 = 8 \times 10^{-8} \text{ m}^4 \) is the cross-sectional moment of inertia. The strain along the beam \( \varepsilon_x(x) \), which is derived by differentiating \( w(x) \) twice with respect to \( x \), varies according to the expression:

\[
\varepsilon_x(x) = \frac{2}{EI} \left[ \frac{P}{2} x - \frac{P}{2} \left( x - \frac{L}{2} \right) - \frac{P}{2} \left| x - \frac{L}{2} \right| \right]
\]

The strain was measured using the above-mentioned single-pulse, phase-coded B-OCDA setup. Figure 46(left) shows the strain measurements due to \( P = 68.6 \text{ N} \) (rectangular markers). The solid line shows a calculation of Eq. (4.19), with bending stiffness term \( EI \) fitted using the least-squared errors method. The analysis results in an estimated stiffness of \( EI = 2590 \text{ Nm}^2 \), or a Young’s modulus of \( E = 32.4 \text{ GPa} \). The calibrated bending stiffness was then reused in Eq. (4.18) for projecting the beam’s deflection curve. Figure 46(right) shows the outcome of this calculation (solid line), alongside the six independently measured deflections (circular markers). Very good agreement was achieved, validating the strain measurements.
The investigation presented herein demonstrated the potential added value of high-resolution Brillouin analysis in composite materials research, and in the development, quality control and maintenance of advanced composite materials products. The technique holds promise for structural health monitoring applications, as well as material science research and development in a wide variety of engineering disciplines.

4.6 Summary

Phase-coded B-OCDA is currently among the leading fiber-optic sensing platforms in terms of the number of resolved points. The method simultaneously provides cm-scale resolution and km-scale range, and allows for the addressing of the entire set of resolution points, as many as hundreds of thousands. Phase coding provides very flexible control over the correlation between the two optical waves, and with it a flexible random access to specific points of interest. Unlike B-OTDA, integration time and resolution may be separated. Unlike previous B-OCDA setups based on frequency modulation, range of unambiguous measurements and resolution may be separated as well. Individual points of interest can be probed for an arbitrarily long time.

In a first experimental demonstration, the Brillouin frequency was measured over a 40 m long fiber, with a spatial resolution of 1 cm, thus addressing 4,000 independent points. The further increase in measurement range was challenging for two reasons: a) each point had to be resolved individually, making the acquisition of
data unrealistically long; and b) the unfavorable scaling of coding noise with the sensing fiber length.

Both restrictions were partially resolved by using a pulsed pump, in what became a combined B-OTDA / B-OCDA approach. In that method, the time-domain amplification of the signal was monitored, resolving the SBS gain in multiple correlation peaks sequentially during the time-of-flight of a single pump pulse. The pulse duration and the phase sequence period were chosen such that individual gain event would be resolved without ambiguity. 80,000 resolution points were successfully addressed along 1.6 km of fiber, with 2 cm resolution. The scheme enables the addressing of the entire set of resolution points using only 127 scans of correlation peaks positions. This number of scans is independent of fiber length. In addition, the combined approach restricted the generation of the off-peak acoustic fields to the spatial extent of the pump pulse, which is much shorter than the fiber length. Coding noise was reduced accordingly.

In another elaboration of the basic principle, multiple pump pulses were launched sequentially into the sensing fiber, thus modulating that wave in two hierarchies: a high-rate, periodic phase code and a slower, aperiodic amplitude code. Incoherent sequence compression was used to distinguish between a large number of extended amplification events that were simultaneously introduced at different correlation peak positions. The protocol allowed for the launch of many pump pulses during a single time-of-flight, increasing OSNR and reducing the acquisition duration accordingly. In an experimental demonstration of the method, an 8.8 km-long fiber was fully scanned with a spatial resolution of 2 cm. The 440,000 points addressed mark the largest number of any Brillouin scattering-based sensor to date. The improvement in number of points, between our first experiment and the last, was more than a hundred-fold in three years. It should be noted again that a similar setup developed by Denisov et al. covered more than 2 million potential resolution points. However, only a subset of these points could be practically addressed.

Lastly, the concept of phase-coded B-OCDA was employed in proof-of-concept experiments of monitoring beams made of a composite material. Optical fibers were
embedded in the beams during their production. The curing process of the epoxy resin was continuously monitored for over 30 hours, showing temperature variations during the exothermic process and a residual strain that remained in the sensing fiber. The excess strain induced by the loading of a beam with external weights was measured and corroborated successfully with predictions made by analytical beam theory. This demonstrations support a promising potential application of high resolution, SBS-based sensors in structural health monitoring and quality assurance.
5 Sensing of Liquids outside Standard Fiber Using Forward Stimulated Brillouin Scattering

5.1 Introduction

One of the most fundamental and widely investigated challenges in fiber sensors research is the analysis of surrounding media [77]. The vast majority of sensing protocols monitor either the refractive index or the absorption spectrum of a substance under test. Such measurements face an inherent difficulty: the analysis mandates some degree of spatial overlap between an optical wave and the medium being studied, whereas light in standard fibers is confined to the core and does not reach the outside of the cladding.

To go around this difficulty, fiber-optic sensors of liquids developed to-date either rely on hollow-core, photonic crystal fibers (PCFs) [78-81], or involve considerable structural modifications of standard fibers. Specific forms of the latter include long-period FBGs for the excitation of cladding modes [82]; etching or polishing the fiber down to the core [83, 84]; tapering fibers down to few-microns diameters [85]; application of transducer coating layers [86]; fabrication of inline cavities [87]; and use of cleaved facets [88, 89]. Many of these devices and setups are extremely sensitive. However, the structural interventions involved in their realization remain a major drawback.

In this chapter we propose and demonstrate the analysis of liquid media outside the cladding of a standard, 8/125 µm single-mode fiber, with no structural modifications or external transducer elements. The measurement principle is based on optomechanics: the stimulation and probing of guided acoustic modes of the fiber structure, at hundreds of MHz frequencies [20, 21, 90-94], by optical waves that are confined to the core. The temporal decay profile of the stimulated acoustic waves is affected by partial reflection and transmission at the outer boundary of the fiber cladding. Therefore, the acoustic impedance of the surrounding medium may be obtained based on acoustic cavity lifetime measurements.
In our experiments we measure the acoustic impedances of deionized water and ethanol. The results agree with values reported in the literature to within less than 1% [95]. Further, the analysis successfully distinguishes between solutions of NaCl dissolved in deionized water, prepared with different salt weight ratios of 4%, 8% and 12%. The dependence of the acoustic impedance on the level of salinity is in agreement with literature as well [96]. These last results illustrate the sensitivity of the analysis protocol, and also its potential application in the monitoring of heavy ionic solutions in industrial processes. The platform represents a conceptual breakthrough in fiber-optic sensing of liquids.

5.2 Principle of Operation

5.2.1 Optical stimulation and probing of guided acoustic waves in standard fibers

5.2.1.1 Radial acoustic modes in standard fibers

Standard optical fibers support several groups of guided acoustic modes [20] [21]. One category is that of radial modes, denoted as \( R_{0,m} \), in which the acoustic field is radially symmetric. Let us denote the radius of the fiber cladding as \( a = 62.5 \mu m \), and the acoustic velocities of longitudinal and shear waves in silica as \( V_l = 5,996 \) m/s and \( V_s = 3,740 \) m/s, respectively. The cutoff frequency of mode \( R_{0,m} \) is given by

\[
 f_{0,m} = \left[ \frac{V_d}{(2\pi a)} \right] y_m ,
\]

where \( y_m \) is the \( m \)th solution to the equation [20]:

\[
 (1 - \kappa^2) J_0(y) = \kappa^2 J_1(y) , \tag{5.1}
\]

and \( \kappa = V_s / V_d = 0.62 \). The transverse profile of density fluctuations in mode \( R_{0,m} \) is proportional to [94]:

\[
 \Delta \rho_{0,m}(r) = \frac{J_0(y_m r / a)}{\sqrt{\int J_0^2(y_m r' / a) r' dr' d\phi}} , \tag{5.2}
\]

where \( r \) is the radial coordinate.
5.2.1.2 Optomechanical stimulation of radial acoustic modes

Figure 47(a) shows a schematic illustration of the dispersion relation between the acoustic frequencies of modes $R_{0,m}$ and their axial wavenumbers $q_{z,m}$ [92]. Immediately above cutoff the modes are entirely transverse, and their group velocity component in the $z$ (axial) direction approaches zero.

For each $m$, there exists a frequency $f_m \approx f_{0,m}$ for which the phase velocity in the axial direction of $R_{0,m}$ matches that of the optical mode in the fiber:

$$2\pi f_m / q_{z,m} = c / n_{eff},$$

where $n_{eff}$ is the effective refractive index of the optical mode (see Figure 47). Radial acoustic modes at these frequencies can couple between two co-propagating optical waves, spectrally detuned by $f_m$. In addition, the modes $R_{0,m}$ may be stimulated by a pair of such optical waves, through electrostriction. The process is illustrated in Figure 47(b). The magnitude of density fluctuations in each radial mode scales with the transverse overlap integral between $\Delta \rho_{0,m}(r)$ and $\nabla_t^2 E^2(r)$, where $E$ denotes the transverse profile of the optical field [92, 93]. While the axial wavenumber components of acoustic waves taking part in such interactions
are nonzero, they are nevertheless much smaller than the corresponding transverse components: \( q_{z,m} \ll 2\pi f_m / V_d \).

Acousto-optic interactions involving radial modes are often referred to as forward stimulated Brillouin scattering (FSBS) [93], guided acoustic wave Brillouin scattering (GAWBS) [20], or Raman-like scattering by acoustic phonons [92]. These interactions are studied extensively since 1985 [20, 21, 90-94]. Experiments show resonant stimulation at frequencies that match very well with the predicted values of \( f_{0,m} \) [20, 21, 90-94]. The modes were mapped in standard single-mode fibers [20], highly nonlinear small-core fibers [93], and PCFs [92]. Scattering by radial acoustic modes was found to limit the propagation length of solitons [97], and distort modulated signals [94]. The temperature dependence of \( f_{0,m} \) was characterized in standard fibers [98], and in PCFs [28-29]. The thermal coefficient of the resonance frequencies in standard fibers is: \( \left( \frac{\partial f_{0,m}}{\partial T} / f_{0,m} \right) = 93 \text{ ppm}/\text{C} \) [98], where \( T \) denotes temperature, the same as that of backwards SBS. The strain dependence of the radial modes frequencies was investigated as well [99].

### 5.2.1.3 Probing the acoustic modes by an optical signal wave

In our experiments, radial acoustic modes are driven by the spectral components of intense, isolated pump pulses at a central optical wavelength \( \lambda_p \). The acoustic vibrations modify the refractive index of the fiber, due to the photo-elastic effect. The resulting index perturbations \( \Delta n \) are very weak [20, 93]. Nevertheless, they may introduce measurable changes in the phase delay of a signal wave, at a different wavelength \( \lambda_s \), that co-propagates with the pump pulses over a distance of few meters or longer [20]. A counter-propagating signal wave, on the other hand, is subject to negligible phase delay variations. The acoustic vibrations therefore introduce a non-reciprocal phase delay to the propagation of the signal wave. The relatively weak signal does not affect the acoustic waves. Kang and coworkers used a Sagnac interferometer loop configuration to convert non-reciprocal phase variations into changes in the intensity of the output signal [92]. We adapt this approach in our sensor platform (see section 5.4.1).
5.2.2 Impedance of surrounding media and the acoustic lifetime

The optical pump wave stimulates an acoustic impulse in the core of the fiber, which represents a superposition of multiple radial modes. The acoustic impulse spreads and radiates outwards from the core, and reaches the outer boundary of the cladding after \(0.5t_r = \frac{a}{V_d}\) seconds (see Figure 48). Part of the acoustic wave magnitude is reflected back towards the core, while another part is transmitted to the surrounding medium.

![Figure 48](image)

Figure 48: Illustrations of the acoustic density variations profile \(\Delta \rho\) as a function of transverse coordinates \(x, y\), following stimulation by a short pump pulse that is confined to the core of a standard fiber. The black lines mark the outer boundary of the fiber cladding. The density fluctuations are a linear combination of the transverse profiles \(\Delta \rho_{0,m}(x, y)\) of individual radial modes \(R_{0,m}\), oscillating at respective frequencies \(f_{0,m}\). Panels (a) through (f) present \(\Delta \rho\) at different instances, noted above each panel. Acoustic impulses form across the core of the fiber at intervals of \(t_r \approx 20.83\) ns.

An echo of the acoustic impulse reforms within the core of the fiber once every \(t_r \approx 20.83\) ns. Each impulse echo is weaker than the previous one, due to transmission losses at the cladding boundary as well as due to internal dissipation within the silica fiber. Therefore, the acoustic field within the core consists of an infinite series of decaying impulses, separated by \(t_r\).

The reflectivity coefficient of the acoustic field magnitude is determined by the acoustic impedance of the silica fiber \(Z_f\) and that if the outside medium \(Z_0\) [100]:

\[
|r_{\text{mir}}| = \left| \frac{Z_f - Z_0}{(Z_f + Z_0)} \right|.
\] (5.3)

The acoustic impedance equals the product of the material density and its acoustic velocity [100]. Impedance values for silica, deionized water and ethanol at
room temperature are $13.19 \times 10^6$, $1.496 \times 10^6$ and $0.953 \times 10^6$ kg/(m$^2$s), respectively [101]. The acoustic impedance of air is much smaller: $416$ kg/(m$^2$s) [101]. It is assumed hereunder that the impedances are independent of the acoustic frequency.

The oscillations lifetime $\tau_m$ of an individual mode $R_{0,m}$ is given by:

$$
\frac{1}{\tau_m} = \frac{1}{\tau_{int,m}} + \frac{1}{\tau_{mir}} + \frac{1}{\tau_r} \ln \left( \frac{1}{|r_{mir}|} \right).
$$

In Eq. (5.4) $\tau_{mir}$ is the lifetime due to transmission losses at the boundary, and $\tau_{m,int}$ denotes the acoustic lifetime associated with internal dissipation, diameter inhomogeneity and ellipticity of the fiber, and all other sources of loss.

Our proposed sensing protocol is therefore the following: with prior knowledge of $\tau_{m,int}$, measurements of the decay time constant of stimulated $R_{0,m}$ modes may recover the acoustic reflectivity coefficient at the cladding boundary, and hence the acoustic impedance of the surrounding medium. The measurements do not rely on the refractive index or absorption spectrum of the medium under test. Furthermore, since both stimulation and probing of the acoustic modes are carried out from within the core, no direct spatial overlap is necessary between optical waves and the analyzed medium.

5.3 Power spectral density of the signal – theoretical model

5.3.1 Acousto-optic excitation of radial modes

Let $E_{pu}(r,z,t)$ denote the electric field of the pump wave, as a function of time $t$, at some given point $z$ along the axis of the fiber and at radius $r$ in the transverse plane:

$$
E_{pu}(r,z,t) = E_0(r)A_{pu}(z,t) \exp \left[ j(\beta_{pu}z - \omega_{pu}t) \right] + \text{c.c.}.
$$

In (5.5), $\beta_{pu}$ and $\omega_{pu}$ are the central wave number and optical frequency of the pump wave, respectively, $A_{pu}(z,t)$ represents its slowly-varying complex
amplitude, and the normalized transverse profile of the optical mode is denoted by $E_0(r)$. The transverse profile is well approximated by a Gaussian shape:

$$E_0(r) = \frac{1}{(2\pi)^{1/2}w} \exp \left( -\frac{r^2}{2w^2} \right),$$  \hfill (5.6)

where $w$ is the Gaussian beam radius. The transverse profile of the optical mode is often stated in terms of its mode field diameter (MFD): $d = 2\sqrt{2}w$ [102]. It is assumed below that the pump wave is undepleted and unaffected by chromatic dispersion or Kerr nonlinearity. Linear losses over the short lengths of fibers under test are neglected as well. Subject to these assumptions, the temporal shape of the pump wave at every position $z$ is simply related to that of the input $A_{pu,in}(t)$, at $z=0$:

$$A_{pu}(z,t) = A_{pu,in}(t - n_{eff,0}z / c).$$  \hfill (5.7)

Here $c$ is the speed of light in vacuum and $n_{eff,0}$ is the effective refractive index of the optical mode.

Consider an amplitude modulated pump, with an instantaneous optical power that is oscillating at radio frequency $\Omega_0 = 2\pi f_0$ with magnitude $|\vec{P}_0|:

$$p_{pu}(t) = |A_{pu,in}(t)|^2 = |\vec{P}_0|^2 + \frac{1}{2}[|\vec{P}_0|^2 \exp(j\Omega_0 t) + \text{c.c.}].$$  \hfill (5.8)

The acoustic waves driven by the pump can be described by the density fluctuations of the medium. The fluctuations can be expressed as a sum of normal modes. The contribution of mode $R_{0,m}$ to the magnitude of density fluctuation is given by:

$$\Delta \rho_m(r,z,t) = \Delta \rho_{0,m}(r) b_{m,0} \exp[j(q_0 z - \Omega_0 t)] + \text{c.c.} .$$  \hfill (5.9)

Here, $\Delta \rho_{0,m}$ is the normalized transverse profile of mode $R_{0,m}$, $b_{m,0}$ denotes its complex amplitude, and $q_0$ is the axial component of its wave number. The acoustic wave is phase-matched with the pump: $\Omega_0 / q_0 = c / n_{eff,0}$. The acoustic amplitude $b_{m,0}$

100
depends on the driving frequency, and on the spatial profile of the acoustic mode. It is maximal when the driving frequency matches the resonance frequency of the mode \( \Omega_{0,m} = 2\pi f_{0,m} \). The dependence of the acoustic wave magnitude on the detuning of the stimulating frequency from resonance follows a Lorentzian lineshape with a width \( \Gamma_m \) [92]:

\[
b_{m,0} = \frac{\varepsilon_o \xi Q_{1,m}}{\Omega_0^2 - \xi^2 + j \Omega_0 \Gamma_m} \hat{P}_0.
\]  

(5.10)

In (5.10), the factor \( Q_{1,m} \) denotes the transverse overlap integral between the profiles of the acoustic mode and the electro-strictive driving force [92]:

\[
Q_{1,m} = \langle \nabla \phi^2, \Delta \rho_{0,m} \rangle \equiv \int_0^2 \int_0^2 \nabla \phi^2 \Delta \rho_{0,m} r dr d\phi.
\]  

(5.11)

Here \( \nabla \phi^2 \) is the transverse component of the Laplacian operator, \( \phi \) is an azimuthal variable, and \( a \) is the outer radius of the fiber cladding.

In our experiments we use a pulsed pump. The instantaneous optical power of the pulse \( P_{pu}(t) \) may be represented in terms of its Fourier components:

\[
P_{pu}(t) = \int_{-\infty}^{\infty} \hat{P}_{pu}(\Omega) \exp(j\Omega t) d\Omega.
\]  

(5.12)

The linear dependence of \( b_{m,0} \) on \( \hat{P}_0 \) allows for the following generalization of (5.9):

\[
\Delta \rho_m(r,z,t) = \Delta \rho_{0,m}(r) h_m \left( n_{eff} z / c - t \right),
\]  

(5.13)

where the time and axial position dependent magnitude \( b_{0,m} \exp(jq_0 z - j\Omega_0 t) \) of (5.9), now becomes \( h_m \left( n_{eff} z / c - t \right) \), with:

\[
h_m(\xi) = \int_{-\infty}^{\infty} \hat{h}_m(\Omega) \exp(j\Omega \xi) d\Omega,
\]  

(5.14)
and:

$$\tilde{h}_m(\Omega) = \frac{\varepsilon\gamma^e Q_{0,m} \tilde{P}_m(\Omega)}{\Omega^2 - \Omega_{0,m}^2 + j\Omega \Gamma_m} . \tag{5.15}$$

### 5.3.2 Optical phase delay of a probe wave due to acoustic perturbations of radial modes

Eq. (5.13) gives the expected density fluctuations for a given temporal shape of the pump pulse. The density variations $\Delta \rho_m$ are accompanied by perturbations in the refractive index $\Delta n_m$, due to the photo-elastic effect [13]:

$$\Delta n_m(r,z,t) = \frac{\gamma_e}{2n_0(r)\rho_0} \Delta \rho_m(r,z,t) . \tag{5.16}$$

Here $\rho_0$ is the density of silica and $n_0(r)$ denotes the transverse profile of the refractive index in the fiber. The relation between local refractive index changes (Eq. (5.16)) and the modification to the overall effective refractive index of the optical mode can be derived using perturbation theory, in the following manner. The transverse profile $F_m(r)$ of an optical mode oscillating at optical frequency $\omega$ must be a solution to the eigenvalue value equation [13]:

$$\nabla_r^2 F_m + k_0^2 n_m^2(r) F = \beta^2_m F_m , \tag{5.17}$$

where $k_0 = \omega / c$ and $n_m(r)$ denotes the transverse profile of refractive index which is given by a combination of $n_0(r)$ and a photo-elastic perturbation due to oscillations of $R_{0,m}$:

$$n_m(r) = n_0(r) + \Delta n_m(r) = n_0(r) + \alpha \tilde{n}_m(r) . \tag{5.18}$$

Here, $\alpha \tilde{n} \equiv \Delta n_m$ is the acoustic disturbance of the $m^{th}$ radial mode, with $\alpha$ a dimensionless scaling factor. Note that typically the photo-elastic contribution to the refractive index is many orders of magnitude smaller than $n_0$ [21]. With that in mind,
we may approximate: $n_m^2(r) \approx n_0^2 + 2\alpha \bar{n} n_0$, and rewrite the eigenvalue equation using the following operator notation:

$$\left( \hat{H}_0 + \alpha \hat{H}_{1,m} \right) F_m = \beta_m^2 F_m, \quad (5.19)$$

where:

$$\hat{H}_0 = \nabla_z^2 + k_0^2 n_0^2(r), \quad (5.20)$$

$$\hat{H}_{1,m} = 2k_0^2 \bar{n} n_0. \quad (5.21)$$

The eigenvalues $\beta_m^2$ and eigenvectors $F_m$ would be those of $\hat{H}_0$ with perturbations due to $\hat{H}_1$. Given that the perturbation is sufficiently weak, they can be written as a power series of $\alpha$:

$$\beta_m^2 = \beta_m^{2(0)} + \alpha \beta_m^{2(1)} + \alpha^2 \beta_m^{2(2)} + \ldots, \quad (5.22)$$

$$F_m = F_m^{(0)} + \alpha F_m^{(1)} + \alpha^2 F_m^{(2)} + \ldots. \quad (5.23)$$

Using standard perturbation theory [13], we find that the zero-order terms are those of $\hat{H}_0$: $\beta_m^{2(0)} = n_{eff,0}^2 k_0^2$ and $F_m^{(0)} = E_0$. The first order correction to the eigenvalue $\beta_m^{2(1)}$ equals:

$$\beta_m^{2(0)} = \left\langle F_m^{(0)}, \hat{H}_{1,m} F_m^{(0)} \right\rangle = 2k_0^2 \left\langle E_0^2, n_0 \bar{n}_m \right\rangle. \quad (5.24)$$

Neglecting higher order terms in $\alpha$, and using (5.16), we may relate the first-order correction to the propagation constant with the magnitude of density fluctuations associated with mode $R_{0,m}$:

$$\beta_m \approx n_{eff,0} k_0 \sqrt{1 + \frac{\gamma e}{n_{eff,0} \rho_0} \left\langle E_0^2, \Delta \rho_m \right\rangle}. \quad (5.25)$$

Noting that $n_{eff} = \beta / k_0$, we obtain the following first-order correction to the effective index of the optical mode due to $R_{0,m}$:
\[ n_{\text{eff}, m}(z, t) \approx n_{\text{eff}, 0} + \frac{\gamma e}{2 n_{\text{eff}, 0} \rho_0} \langle E^2_0, \Delta \rho_m \rangle. \quad (5.26) \]

Substituting \( \Delta \rho_m \) from Eq. (5.13), we obtain:

\[ \Delta n_{\text{eff}, m}(z, t) \equiv n_{\text{eff}, m} - n_{\text{eff}, 0} = \frac{\gamma e Q_{0,m}}{2 n_{\text{eff}, 0} \rho_0} h_m \left( t - n_{\text{eff}, 0} z / c \right), \quad (5.27) \]

With:

\[ Q_{0,m} \equiv \langle E^2_0, \Delta \rho_{0,m} \rangle = \int_0^{2\pi} \int_0 \int_0 ( \hat{E}^2_0 \Delta \rho_{0,m} r \, dr \, d\phi). \quad (5.28) \]

### 5.3.3 Interferometric measurements of acoustic waves

In the experimental procedure described here (section 5.2.1.3), a Sagnac loop interferometer is used to measure the non-reciprocal phase delay \( \Delta \phi \) induced by optically-stimulated radial acoustic modes. The optical power of a probe wave at the loop output \( P_{\text{det}} \), when the loop is balanced and biased at quadrature and \( \Delta \phi \ll \pi \), may be expressed as:

\[ P_{\text{det}}(t) \approx P_{\text{det,dc}} \left[ 1 + \Delta \phi(t) \right]. \quad (5.29) \]

Here \( P_{\text{det,dc}} \) is a bias power level which is obtained in the absence of non-reciprocal phase delay. The non-reciprocal phase, in-turn, is determined by the photoelastic variation in the refractive index induced by the acoustic density fluctuations of all modes:

\[ \Delta \phi(t) = k_v \int_0^L \sum_m \Delta n_{\text{eff}, m} \left( L - z, t - n_{\text{eff}, 0} z / c \right) dz, \quad (5.30) \]

with \( L \) denoting the length of the fiber. Substituting (5.27) into (5.30), we have:

\[ \Delta \phi(t) = \frac{k_v \gamma e L}{2 n_{\text{eff}, 0} \rho_0} \sum_m Q_{0,m} h_m \left( t - n_{\text{eff}, 0} L / c \right). \quad (5.31) \]
Taking the Fourier transform of the non-reciprocal phase, we obtain the following expression for the spectral components of the detected power at any $\Omega \neq 0$:

$$
\tilde{P}_{\text{det}}(\Omega) = \int P_{\text{det}}(t) \exp(-j\Omega t) \, dt = \frac{P_{\text{det,dc}} k_0 \gamma_e^2 c_e L}{2n_{\text{eff,0}} \rho_0} \sum_m Q_{0,m} \tilde{h}_m(\Omega) .
$$

(5.32)

Let us define the constant:

$$
C_1 \equiv \frac{P_{\text{det,dc}} k_0 \gamma_e^2 c_e L}{2n_{\text{eff,0}} \rho_0} ,
$$

(5.33)

for brevity. The power spectral density (PSD) of the detected power is given by substituting (5.15) into (5.32):

$$
|\tilde{P}_{\text{det}}(\Omega)|^2 = C_1^2 \sum_m \frac{|Q_{0,m}|^2 |Q_{1,m}|^2 |\tilde{P}_{pu}(\Omega)|^2}{\Omega^2 \left[ 4(\Omega - \Omega_{0,m})^2 + \Gamma_m^2 \right]}
$$

(5.34)

In (5.34) a negligible spectral overlap between $\tilde{h}_{m1}(\Omega)$ and $\tilde{h}_{m1}(\Omega)$ was assumed for any two integers $m1 \neq m2$. Similarly, the value of the PSD on a specific resonance $m = M$ is only affected by the contribution of $R_{0,m}$:

$$
|\tilde{P}_{\text{det}}(\Omega_M)|^2 = C_1^2 \frac{|Q_{0,M} Q_{0,M}|^2}{\Omega_M^2 \Gamma_M^2} |\tilde{P}_{pu}(\Omega_M)|^2
$$

(5.35)

### 5.4 Experimental Results

#### 5.4.1 Experimental setup

A schematic illustration of the experimental setup is provided in Figure 49 [92, 103]. Light from a laser diode source ($\lambda_p = 1560$ nm) was amplitude-modulated in a semiconductor optical amplifier (SOA), driven by repeating current pulses of 20 ns duration and 20 $\mu$s period. The extinction ratio of pulses at the SOA output was higher than 30 dB. The pump pulses were further modulated to 1 ns duration with the same period, by an EO Mach-Zehnder amplitude modulator. The pump wave was amplified by an EDFA, and launched into a section of standard, single-mode fiber under test.
Due to the low duty cycle and high extinction ratio of the SOA modulation, high peak power levels of pump pulses could be obtained at the EDFA output. The peak power levels were adjusted to 4.3 W. The temporal shape of the pump pulse $P_{pu}(t)$ and its PSD $|\tilde{P}_{pu}(\Omega)|^2$ are presented in Figure 50.

---

**Figure 49:** Schematic illustration of the experimental setup used in the stimulation and probing of radial guided acoustic modes in standard fibers. SOA: semiconductor optical amplifier. BPF: bandpass filter.

The FUT was placed within a Sagnac loop ([92, 103], Figure 49). A signal wave from a second laser diode at $\lambda = 1550$ nm was launched into the loop in both directions. A polarization controller (PC) within the loop was used to bias the loop output at quadrature. In the absence of dynamic non-reciprocal phase disturbances, the signal power was split equally between the two output ports of the loop. The phase delay of the clockwise-propagating signal wave was modified by radial acoustic modes stimulated by the pump pulses, whereas the counter-clockwise propagating signal waves were subject to much smaller phase perturbation. For sufficiently small differential phase delays $\Delta \phi \ll \pi$, variations in the signal power at the loop output were linearly proportional to $\Delta \phi$. 
An optical bandpass filter (BPF) was connected to the FUT within the loop, at the output end of the pump wave. The BPF allowed the signal wave to propagate in both directions, while the pump pulses were blocked from reaching the loop output. The output signal wave was detected by broadband photo-detector and sampled by a real-time digitizing oscilloscope at 1 GHz bandwidth. The sampled traces were averaged over 8,192 repeating pump pulses, and analyzed using offline signal processing.

The pump pulses may also stimulate torsional-radial acoustic modes in the fiber, denoted as $TR_{2,m}$ [104]. The resonant spectra of these modes overlap those of $R_{0,m}$ modes, and their excitation interferes with the data analysis. The $TR_{2,m}$ modes introduce birefringence and polarization-dependent scattering of the signal wave [104]. In order to suppress the contribution of these modes, a polarization scrambler was placed along the input path of the pump wave. The effect of the $TR_{2,m}$ modes on the output signal wave was canceled out of the averaged output trace.
5.4.2 Results: stripped fiber in deionized water, ethanol and air

In a first set of experiments, a 30 meters long section of FUT was stripped of its polymer coating using sulfuric acid. Figure 51(top) shows an example of the signal output trace $V(t)$ as a function of time $t$, with the FUT exposed in the air. A magnified view of the first 200 ns of the output trace is shown in Figure 51(bottom). The first 10 ns of the trace were disregarded due to residual leakage of the intense pump pulses and cross-phase modulation effects (not shown). A series of impulses separated by $t$, is observed, as expected.

![Figure 51: (top) – Measured power of the signal wave as a function of time, at the output of the Sagnac interferometer loop. The fiber under test was stripped of its polymer coating and kept exposed in the air. (bottom) – Magnified view of the top panel in its first 200 ns. A decaying sequence of impulses, separated by ~20.83 ns, is observed.](image)

The power spectral density $|V(f)|^2$ of the output trace is presented in Figure 52(a). Multiple resonances corresponding to the $R_{0,m}$ modes are observed. Table 3 lists the experimentally obtained frequencies $f_{0,m}$ alongside the predicted values (Eq. (5.1), [20, 93]). Excellent agreement is achieved. The calculated frequencies are
consistently higher than the predicted values by $0.7 \pm 0.15\%$. This difference possibly suggests a cladding diameter of $125.8 \, \mu m$, which is within specified tolerances.

Figure 52(b) presents the measured relative power in each mode $|\mathcal{V}(f_{0,m})|^2$ alongside the expected modal strengths, calculated based on the transverse profiles of the acoustic modes and the optical mode and the power spectral density of the pump pulses (see section 5.3, Eq. (5.35)). The overlap integrals $Q_{0,m}, Q_{1,m}$ were evaluated numerically according to (5.11) and (5.28). The transverse profile of the optical mode is provided in Eq. (5.6) and those of the radial modes are given by (5.2). A standard single-mode fiber with a MFD of $10.4 \, \mu m$ was considered in the calculation. The linewidths $\Gamma_m$ were calculated as the full-widths at half maximum (FWHM) of the observed spectral lines. Experimental error among consecutive measurements of modal strength is bound by $\pm 5\%$. Good quantitative agreement is found between model and measurements (Table 3). The stimulation of the lowest-order modes $m=1,2$ is inefficient due to the small overlap between their broad transverse profiles and that of the electro-strictive driving force. The stimulation of modes $m \geq 13$ is limited by the bandwidth of pump pulses.
Figure 52: (a) - Power spectral density of the measured output signal trace. The spectrum consists of a series of discrete modes, whose frequencies match the cutoff frequencies of the radial acoustic modes (see Table 3). (b) - Measurement and calculation of the maximum power spectral density in each mode.

Table 3: Measured and calculated frequencies and relative power levels of guided radial acoustic modes in a standard fiber.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured freq. (MHz)</th>
<th>Calculated freq. (MHz)</th>
<th>Measured rel. power</th>
<th>Calculated rel. power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.34</td>
<td>30.53</td>
<td>0.00084</td>
<td>0.00016</td>
</tr>
<tr>
<td>2</td>
<td>81.66</td>
<td>82.07</td>
<td>0.0048</td>
<td>0.007</td>
</tr>
<tr>
<td>3</td>
<td>130.1</td>
<td>130.8</td>
<td>0.0282</td>
<td>0.0282</td>
</tr>
<tr>
<td>4</td>
<td>178.1</td>
<td>179.0</td>
<td>0.049</td>
<td>0.0546</td>
</tr>
<tr>
<td>5</td>
<td>226.0</td>
<td>227.2</td>
<td>0.1127</td>
<td>0.1106</td>
</tr>
<tr>
<td>6</td>
<td>273.3</td>
<td>275.3</td>
<td>0.1545</td>
<td>0.1509</td>
</tr>
<tr>
<td>7</td>
<td>321.4</td>
<td>323.3</td>
<td>0.183</td>
<td>0.1752</td>
</tr>
<tr>
<td>8</td>
<td>369.1</td>
<td>371.4</td>
<td>0.1575</td>
<td>0.1574</td>
</tr>
<tr>
<td>9</td>
<td>416</td>
<td>419.4</td>
<td>0.1247</td>
<td>0.1247</td>
</tr>
<tr>
<td>10</td>
<td>464</td>
<td>467.3</td>
<td>0.0849</td>
<td>0.091</td>
</tr>
<tr>
<td>11</td>
<td>511</td>
<td>515.4</td>
<td>0.0547</td>
<td>0.0544</td>
</tr>
<tr>
<td>12</td>
<td>559</td>
<td>563.3</td>
<td>0.0299</td>
<td>0.0305</td>
</tr>
<tr>
<td>13</td>
<td>607</td>
<td>611.3</td>
<td>0.0149</td>
<td>0.0154</td>
</tr>
</tbody>
</table>
Figure 53(a) shows $V(t)$ with the FUT immersed in deionized water and in a mixture of 95% ethanol and 5% methanol. Compared with measurements with the FUT in air, the series of impulses in these traces decay much more quickly. The traces were digitally filtered to select contributions of individual modes, $m=1$ to 13. Figure 53(b) shows the filtered traces for mode $m=5$. The modal oscillations are well characterized by an exponential decay, with a single time constant for each mode. The decay is dominated by acoustic transmission losses at the outer boundary of the fiber cladding. The lifetimes of individual modes $\tau_m$, with the FUT in deionized water and ethanol, are shown in the lower two curves of Figure 53(c).

The assessment of acoustic reflectivity $|r_{mir}|$ at the cladding boundary requires knowledge of the intrinsic losses of the acoustic modes (Eq. (5.4)).
\[ |r_{mr}| = \exp \left( -\frac{\tau_{int,m} - \tau_m}{\tau_{int,m} \cdot \tau_m} t_r \right). \quad (5.36) \]

The intrinsic lifetimes \( \tau_{int,m} \) were estimated based on measurements taken with the FUT in air. Oscillations of individual modes were filtered as described above. An example for \( R_{0.5} \) is shown in Figure 54. Due to the low acoustic impedance of air, reflectivity at the cladding boundary practically equals unity for all modes. The acoustic resonances with the FUT in air are broadened by inhomogeneity in cladding radius along the fiber length [93], and hence their temporal decay cannot be described by a single time constant. Effective intrinsic lifetimes were therefore estimated based on the first 150 ns of the filtered traces, a duration that is comparable with modal lifetimes observed with the FUT immersed in test liquids. The estimated lifetimes do not fully describe the entire dynamics of modal oscillations with the fiber in air. Nevertheless, they account for the acoustic losses mechanisms, other than transmission at the boundaries, within the time frame that is relevant to the analysis of liquids. The estimates of \( \tau_{m,\text{int}} \) are shown in the upper trace of Figure 53(c).

![Figure 54: Measured oscillations of acoustic mode \( R_{0.5} \) at 226 MHz with the fiber under test in air, obtained by digital filtering of the trace of Figure 53(a).](image)

Figure 55 presents the experimentally obtained acoustic impedances of deionized water and ethanol, respectively, using the time constants of modes \( m=3 \) to 11. Results were averaged over 30 consecutive experiments. The acoustic impedances are independent of frequency, in support of previous assumptions. The experimental impedances are \( 1.475e6 \pm 0.01e6 \) kg/(m²s) for deionized water, and
$0.952 \times 10^6 \pm 0.03 \times 10^6$ kg/(m$^2$/s) for ethanol. The corresponding values reported in the literature are $1.485 \times 10^6$ kg/(m$^2$/s) and $0.953 \times 10^6$ kg/(m$^2$/s), respectively [95]. The experimental uncertainties represent the differences between impedance estimates based on different modes. Impedance variations among repeating analyses of a given medium using an individual mode are below 0.1%. Measurements and reference data agree to within less than 1%. The results demonstrate that the fiber-sensor setup can quantitatively measure the acoustic impedance of surrounding liquids outside the cladding.

\[
\begin{array}{c}
\text{Impedance [kg/(m}^2\text{s)]} \\
\hline
\text{Mode frequency [MHz]} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Water} \\
\text{Ethanol} \\
\end{array}
\]

**Figure 55**: Solid lines: Measured acoustic impedances of deionized water (blue) and ethanol (red), as a function of frequency of acoustic radial modes. Dashed line: corresponding reference acoustic impedance values [101].

**5.4.3 Results: fiber in solutions of dissolved salt in deionized water**

The acoustic impedance of aqueous solutions is known to change with the concentration of dissolved salts [96]. In a second set of experiments, the FUT was immersed in several solutions of deionized water with 4%, 8% and 12% relative weight of NaCl. The acoustic impedances of the solutions were recovered using the experimental procedure described in the previous subsection. The results are plotted in Figure 56. The measured impedances of deionized water are shown again for comparison. Results are summarized in Table 4.
Figure 5.5: Measured acoustic impedances of solutions of deionized water, as a function of frequency of acoustic radial modes. Colors denote aqueous solutions with dissolved NaCl at relative weight ratios of 0, 4%, 8% and 12% (see legend).

Table 4: Measured acoustic impedance of liquids under test, and corresponding reference values (in units of 1e6 kg/m²s).

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Measured impedance</th>
<th>Reference impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethanol</td>
<td>0.952 ± 0.03</td>
<td>0.953 [95]</td>
</tr>
<tr>
<td>Deionized water</td>
<td>1.475 ± 0.01</td>
<td>1.485 [95]</td>
</tr>
<tr>
<td>Water + 4% NaCl</td>
<td>1.524 ± 0.03</td>
<td>1.571 [96]</td>
</tr>
<tr>
<td>Water + 8% NaCl</td>
<td>1.572 ± 0.03</td>
<td>1.664 [96]</td>
</tr>
<tr>
<td>Water + 12% NaCl</td>
<td>1.67 ± 0.03</td>
<td>1.763 [96]</td>
</tr>
</tbody>
</table>

The measurements clearly distinguish between the four solutions. The acoustic impedance is seen to increase with the level of salinity, in agreement with expectations. Quantitative agreement between the results and previously stated values is partial: we observed 1.2% increase in impedance per 1% increase in the relative weight of dissolved salt, as opposed to a corresponding impedance change by 1.6% reported in the literature ([96], see Table 4). The reasons for this disagreement are still under study. Despite these differences, the results illustrate the ability of the sensor setup to resolve fine changes in acoustic impedance.

5.5 Conclusions

In this chapter, we proposed and demonstrated an optomechanical fiber sensor that is able to measure the acoustic impedance of a liquid medium outside the cladding of standard single-mode fibers. The outer boundary of the cladding is probed by guided radial acoustic modes of the fiber structure. Both the stimulation and the
monitoring of acoustic modes are carried out by optical waves that are confined to the core. Unlike previous sensors of liquids, no structural modification of the standard fiber was necessary. The results represent a new paradigm towards one of the basic challenges of fiber-optic sensors research.

The impedances of deionized water and ethanol were measured, in excellent agreement with literature values. The analysis also identified fine changes between the impedances of aqueous solutions with different concentrations of dissolved salt. The proposed sensor platform could be applied to the monitoring of industrial processes involving heavy ionic solutions, such as in electro-chemistry or desalination of water.

Stronger pump pulses would support the use of shorter FUTs, towards meter-scale. Note however that the pump path within the Sagnac loop inevitably contains residual segments of coated fibers, leading to components such as couplers or BPF. The acoustic characteristics of these sections should be taken into account in the analysis of data collected over very short FUTs. Finally, the measurements currently rely on the interaction between co-propagating pump and signal waves, and does not support distributed analysis. Future research would concentrate on trying to remove this limitation.
6 Summary and Conclusions

6.1 Summary

6.1.1 Distributed fiber sensors based on phase-coded B-OCDA

In this dissertation I have reported on several innovations in the field of distributed fiber sensing based on SBS. The first breakthrough of the presented research has been the generalization of the B-OCDA technique to the joint, high-rate binary phase coding of both pump and signal. This allowed for better control over the cross-correlation between the two waves, and for a high degree of flexibility in controlling the correlation peaks locations and spatial extent. The technique also removed the tradeoff between spatial resolution and range of unambiguous measurement which characterizes previous, frequency modulation based B-OCDA setups. A specific class of binary codes, the perfect Golomb codes, proved to be advantageous in reducing coding noise. Phase coding enabled a random-access, high-resolution Brillouin analysis. An experimental demonstration of the initial principle included the analysis of a 40 m-long fiber with 1 cm spatial resolution, sequentially addressing all 4,000 points.

Further variations on the theme included the probing of multiple resolution points in a single round-trip propagation of the pump wave through the fiber. By intentionally using a repeating phase code with a short period, multiple correlation peaks were introduced along the fiber. The SBS gain in each correlation peak was separated by time domain analysis, in what became a combination of B-OCDA and B-OTDA principles. The hybrid approach brought together the high resolution capabilities of B-OCDA and the reduced acquisition duration of B-OTDA. In addition, the extent of coding noise was much reduced. A 1600 m-long fiber was analyzed with 2 cm resolution. All 80,000 points were successfully addressed using only 127 scans of correlation peaks positions.

In a later, more involved technique, a series of pump pulses was launched into the fiber, and the multiple, temporally-extended and overlapping SBS gain events were separated via an incoherent sequence compression protocol. A single trace in
this measurement provided a potential improvement in OSNR that was equivalent to
the averaging over thousands of repetitions of single-pulse experiments. Therefore,
the experimental duration could be reduced even further. This duel-hierarchy coding
technique of both phase and amplitude was demonstrated experimentally in the
analysis of an 8.8 km long fiber with 2 cm spatial resolution. The number of addressed
points was increased by two orders of magnitude during the span of this research,
from the initial 4,000 points to 440,000 points.

In the experiment, a polarization scrambler was used in order to avoid
polarization-induced fading of the SBS interaction. Polarization scrambling is a
comparatively slow process, with a characteristic time scale of 1 µs. The sequence
compression protocol is effective at reducing noises with coherence lengths shorter
than the modulation symbol duration, which is in the order of 20 ns. Hence, additional
averaging over multiple traces was necessary to reduce the scrambling-induced noise,
which partly took away the advantage gained by using long pulse sequences. In
practice, the time required to complete a scan of the fiber under test was considerably
longer than the theoretical lower bound (see Eq. (4.16)), so that the full capabilities of
the system have not yet been reached. Use of recently proposed, more advanced
polarization switching and diversity schemes may further reduce the experimental
duration and perhaps enable the addressing of even larger numbers of points.

Another practical limitation in our experiments had to do with the very long,
fixed delay lines that were necessary for the spatial scanning of correlation peaks
positions. The long delay line imposes a high degree of temperature sensitivity, which
must be compensated for in order to assure cm-scale accuracy in the locations of the
correlation peaks. As demonstrated by members of our group, thermal drifts may be
effectively removed in a closed-loop operation, based on chromatic dispersion and
wavelength tuning of the laser source [105].

A more fundamental limitation stems from residual coding noise. Use of
extended pulse sequences as the pump wave leads to the generation of residual off-
peak acoustic waves along the entire fiber under test, like in the initial B-OCDA
concept. Although coding noise is strongly suppressed by the incoherent compression
protocol, it does not vanish entirely. The quantitative study of coding noise in the dual-hierarchy modulation scheme is ongoing. However, coding noise is already identified as a performance limiting factor.

Compared with the state-of-the-art of current SBS based distributed sensors, our results stand out in terms of the number of overall addressed points during a single experiment. High resolution can be achieved by FM B-OCDA [6] or by ASE based B-OCDA [106], down to mm-scale. Long range measurements relying on Raman amplification and inline EDFAs can reach up to 600 km of range [28]. High range and high resolution may be achieved by using the DPP B-OTDA method, with ranges of up to 60 km and spatial resolution of 25 cm [107]. In a method similar to the combined B-OTDA/B-OCDA described here, more than 2 million potential resolution points could be arbitrarily addressed, over a 17.5 km long fiber with 8 mm spatial resolution [65]. However, to the best of our knowledge, the 440,000 points addressed with the dual-hierarchy, phase and amplitude coding approach described in this work represent the largest set of data points of any Brillouin sensor.

6.1.2 DBGs based on phase-coded SBS

Phase-coded SBS interactions can effectively establish a localized and time-stationary acoustic grating. Chapter 3 described the generation of such gratings by two phase-coded pump waves that were polarized along one principal axis of a PM fiber. These gratings reflected probe waves that were polarized along the orthogonal principal axis and were properly detuned in frequency. This arrangement, referred to as a phase-coded DBG, supports a broad reflectivity spectrum that is governed by the modulation rate of the phase code.

One application of the phase-coded DBGs was in the long, variable all-optical delay of probe waveforms. The delay of 1 ns pulses by as much as 770 ns was demonstrated, however the accumulation of coding noise mandated averaging over multiple repeating acquisition. Coding noise can be reduced by a factor of 3-4 with the use of perfect Golomb codes in the phase modulation. With the use of these codes, the delay of 1 Gb/s, NRZ PRBS probe data by as much as 10 ns has been achieved, with sufficiently high SNR to obtain an open eye diagram. Good agreement was found
between the experimental results and simulations. The analysis suggests that further improvement in bandwidth and OSNR is expected for higher phase modulation rates, at the expense of a lower reflectivity and higher setup complexity.

The suppression of other noise mechanisms in the variable delay setup is challenging as well. The reflectivity of a 2-cm long DBG is very low, on the order of -60 dB. High-power EDFAs were therefore required for both pumps, as well as for the input signal and the reflected echo. This arrangement is not always appropriate in commercial applications, and may restrict the concept to the realm of academia. Nevertheless, the delay-bandwidth product achieved by the phase-coded DBG is an order of magnitude higher than that obtained using broadband SBS slow light-based setups, for example [56]. The setup also involves a minimal wavelength offset, as opposed to variable delays that are based on chromatic dispersion [108, 109].

6.1.3 Sensing of liquids outside the cladding of standard fibers

In Chapter 5 of this work, we reported on a new approach to one of the most fundamental and long-standing challenges in fiber-optic sensors research - the analysis of liquid media. Existing sensor architectures require spatial overlap between light and the substance being tested, and rely either on structural modifications of standard fibers or on specialty photonic crystal fibers. In contrast, we used standard 8/125 µm single-mode fibers with no structural intervention. Radial acoustic modes, supported by the cylindrical shape of the fiber, were pumped and probed by optical waves propagating in the core of the fiber. The cavity life-time of each excited acoustic mode was analyzed to provide the acoustic impedance of the surrounding medium.

We measured the acoustic impedances of deionized water and ethanol, in excellent agreement with literature values. Measurements could also distinguish between the impedances of aqueous solutions containing 0, 4%, 8% and 12% weight ratios of dissolved salt. Lastly, a theoretical model for the PSD of a signal wave, which is perturbed by optically-stimulated radial acoustic modes, was proposed and validated in very good agreement with experiments.

Quantitative agreement between the measured acoustic impedances of aqueous solutions of NaCl and literature values was only partial. We observed 1.2%
increase in impedance per 1% increase in the relative weight of dissolved salt, as opposed to a corresponding impedance change by 1.6% reported in the literature. Further study is required to clarify this discrepancy.

The proposed measurement system may lead to a new paradigm in the fiber-optic sensing of liquids. An important stepping-stone towards this goal would be to shorten the length of fiber involved. Here, we used a 30 m long fiber. Use of fibers that are only a few meters long is possible, but would require higher pump power levels. In addition, the measurement setup inevitably involves the stimulation of acoustic waves in short segments of coated fiber, such as those leading to various components. These must be taken into account in the data analysis. The effect of coated segments becomes more pronounced when the test fiber itself becomes shorter. Lastly, the current setup relies on co-propagating pump and signal waves and does not support distributed measurements. The future removal of this limitation, if possible, would represent very significant advance.

### 6.2 Extensions and future prospects

#### 6.2.1 B-OCDA based on amplified spontaneous emission

The resolution of any B-OCDA setup depends on the coherence length of the pump and signal waves. In the case of phase coding presented in this dissertation, as well as in frequency modulation-based B-OCDA [6], the coherence length is determined by the rate of modulation. That rate, in turn, is limited by the instruments available. In a different approach to B-OCDA [106], the pump and signal waves are extracted from a common ASE source, and filtered by a BPF so that the width of the BPF governs their coherence length. Since an optical BPF can be several nm-wide, the spatial resolution can be, in principle, below 1 mm [106].

Although the measurement of the very weak gain obtained over such short segments of fiber can be practically challenging, a spatial resolution of 4 mm was successfully demonstrated using this technique. The SNR of the measurements may be further improved with more careful separation between pump and probe upon detection. Ultra-high resolution Brillouin analysis may be instrumental in precision
measurements of strain, geometry and mode evolution in tapered fibers, couplers, more complex fiber-based components and even waveguides.

6.2.2 Transient analysis in B-OCDA

In most SBS-based sensing techniques reported to date, the signal gain is measured at the steady state. One exception is slope-assisted Brillouin analysis [29, 110-112]. In this method the frequency difference between pump and signal is deliberately set to either side of the Brillouin frequency shift, where the variations in signal gain with frequency offset are the steepest. With that choice, changes in the Brillouin frequency modify the intensity of the output signal. Slope-assisted B-OTDA is successfully employed in high-rate dynamic monitoring of vibrations. The technique is very effective in the identification of disturbances, however the quantitative measurement of the Brillouin shift is often restricted to a comparatively narrow dynamic range. In addition, slope assisted measurements are sensitive to intensity noise and often require prior mapping of the 'nominal' Brillouin shift at each position.

In a new, alternative approach followed by our group, it is proposed and demonstrated that the transient response of the signal gain, as it begins the interaction with a pump pulse, contains information regarding the frequency detuning with respect to the Brillouin shift [113]. A single scan, taken at an arbitrary $\Omega$, may still resolve the correct Brillouin shift of the fiber. A dynamic range of 200 MHz was obtained in preliminary experiments. This scanning-free approach can potentially reduce the data acquisition time by an order of magnitude.

Initial experiments were carried out over 2 m of fiber, with no distinction among different locations. However, the combination between phase-coded, time-gated B-OCDA as described in this work and transient analysis may provide high resolution measurements that are free of spectral scanning. Such integration is the subject of an ongoing study.

6.2.3 Double pulse pair B-OCDA

The DPP B-OTDA method was described in section 1.4.1. It is a form of B-OTDA, in which the time-domain amplification of a single pump pulse is subtracted from the gain of a second, slightly longer pulse. The difference trace effectively resolves the SBS
gain over short sections of fiber, corresponding to the difference between the widths of the pair of pulses. These setups have reached cm-scale resolution over few km, and they address the entire length of the fiber under test with only two scans per choice of $\Omega$. On the other hand, they require pulses with very sharp edges and a broad measurement bandwidth.

Now, consider the combined B-OTDA/B-OCDA method proposed in this work. Suppose that the DPP protocol was used in this setup: repeating measurements with a pair of pulses, one slightly longer than the other. In both acquisitions the same short phase code will be used to repeatedly modulate the pump pulse and the continuous signal, so that both traces will address the same set of discrete correlation peaks. The duration of each pump pulse will be longer than the phase code period, (see Eq. (4.8) ), hence multiple correlation peaks will be simultaneously introduced and the analysis of each single trace would be ambiguous. However, the difference between the two traces would be able to resolve the SBS amplification taking place at individual correlation peaks.

This protocol may provide several advantages. First, the separation between adjacent peaks will no longer be limited by the acoustic lifetime. This may allow for the use of much shorter phase codes, and reduce the number of position scans that is necessary to address the entire fiber accordingly. In addition, the difference in duration between the two pump pulses may be considerably longer than that of DPP B-OTDA, since the spatial resolution is governed by the phase code rate and not by that difference. The necessary measurement bandwidth may be relaxed accordingly. A DPP, phase-coded B-OCDA setup would represent an intermediate working point between DPP B-OTDA (a broad bandwidth scheme involving only two scans) and the current hybrid B-OTDA / B-OCDA setup of this work (narrow bandwidth detection involving typically 100 positions scans).

### 6.2.4 Impact of Laser phase noise on OSNR

One interesting and potentially detrimental aspect of phase coded SBS that was not explored during this research is the potential impact of laser phase noise. In the analysis given in this document it is assumed that the phase of the electric field
(e.g. in Eq. (1.15)) is perfectly stable. In practice, the phase of a laser is stochastic in nature and may change significantly within time frames that exceed its coherence time. Such variations may affect the amplitude of the acoustic field at a correlation peak, in the same manner that phase modulation suppresses the correlation side-lobes, thus degrading the OSNR. The consequences of this effect on the performance of B-OCDA sensing and DBGs are yet to be studied.

6.2.5 Study of guided acoustic waves Brillouin scattering in multi-core fibers

In this study, acoustic modes of standard single mode fibers were stimulated and monitored. The effective excitation of these modes through electrostriction and the consequential phase modulation of signal waves require non-vanishing spatial overlap between the acoustic and the optical modes. The radial symmetry of standard fibers restricts the classes of acoustic modes that may be stimulated and probed to only two sets: the radial, $R_{0m}$ modes used in this work and the torsional-radial $TR_{2m}$ modes.

In contrast, multi-core fibers contain several cores within the 125 µm cladding. Some of the cores are located away from the center of the cladding, breaking the radial symmetry. In theory, pump waves propagating in an off-axis core should excite classes of acoustic modes that thus far could not be observed with standard single-core fibers. Multi-core fibers are intended for use in optical space-division multiplexing [114]. Guided acoustic waves of the fiber cladding may introduce crosstalk between cores. The study of guided acoustic waves in multi-core fiber may therefore be of both fundamental and practical consequences.
References


[64] C. Zhang, M. Kishi, and K. Hotate, "Enlargement of measurement range in Brillouin optical correlation domain analysis with high-speed random accessibility using temporal gating scheme for multiple-points dynamic strain"


Appendix – Golomb codes

The Golomb codes used in this work are defined as follows. All elements in the code are of unity magnitude. For a code of length \( M_p \), the phases of elements \( c_n \) where \( n \in \mathbb{C}_{M_p} \) equal \( \cos^{-1}\left[\left(1-M_p \right) / \left(1+M_p\right)\right] \). The phases of all other elements equal zero [115]. The codes lengths \( M_p \) and sets \( \mathbb{C}_{M_p} \) are listed in Table 5.

<table>
<thead>
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<th>( M_p )</th>
<th>( \mathbb{C}_{M_p} )</th>
</tr>
</thead>
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</tr>
<tr>
<td>83</td>
<td>{ 2 5 6 8 13 14 15 18 19 20 22 24 32 34 35 39 42 43 45 46 47 50 52 53 54 55 56 57 60 62 66 67 71 72 73 74 76 79 80 82 83 }</td>
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<tr>
<td>127</td>
<td>{ 3 5 6 7 10 12 14 20 23 24 27 28 29 33 39 40 42 43 44 45 46 48 51 53 54 55 56 57 58 59 63 65 66 67 75 77 78 80 83 85 86 89 90 91 92 93 95 96 97 101 102 105 106 108 109 110 111 112 114 116 118 119 123 125 126 127 }</td>
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<tr>
<td>211</td>
<td>{ 1 3 4 8 9 11 13 16 18 19 23 24 27 28 29 30 32 33 34 36 39 40 41 42 43 49 51 58 61 62 64 68 69 73 75 76 78 86 87 89 90 91 92 93 95 98 99 103 105 107 109 111 112 113 116 117 119 125 128 129 130 131 132 133 134 136 139 141 142 143 146 147 148 150 153 154 156 157 158 159 160 161 163 165 166 167 168 169 175 176 178 182 187 188 191 192 193 196 198 199 201 203 206 207 208 211 }</td>
</tr>
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</table>
Stimulated Brillouin Scattering (SBS), is an optoacoustic phenomenon. It is a non-linear interaction between two optical waves, called pump and signal, and an acoustic wave, which lies between the two. The phenomenon was named after the French physicist Léon Nicolas Brillouin and was first observed by Chiao and colleagues in 1964. In the late 1990s, Horiguchi demonstrated that by using the SBS interaction, it is possible to perform a backscatter measurement over standard fiber lengths. The technique is based on the dependence between the oscillation frequency of the phenomenon and the physical properties of the fiber, especially temperature and curvature. By limiting the length of the pump to short pulses, it is possible to extract information about the local interaction by analyzing the temporal growth of the signal.

This technique is known as Brillouin optical time domain analysis (B-OTDA). Typical B-OTDA systems can analyze dozens of kilometers of fiber, but they are limited in resolution to the order of a few meters.

Since the late 1990s, many researches, both academic and industrial, have been dedicated to improving the capabilities of fiber-based SBS sensors, in terms of resolution, range, precision, and speed.

One of the ways is Brillouin optical correlation analysis (B-OCDA) which was developed first by Prof. Hotate in 2000. The method is based on the relationship between the length of the optical wave and the autocorrelation of the superimposed waves. In the initial version, the amplitude of the 

pump and signal is constant, while their frequency is shared by a sinusoidal wave. As a result, the two waves remain synchronized at discrete points, known as correlation peaks.

Therefore, the interaction is mainly limited to those points, which can be a few millimeters wide. A precise measurement requires that only one correlation point is created along the fiber. Due to the periodic nature of the sinusoidal wave, the measurement distance was initially limited to a few meters. Mapping the SBS interaction along the fiber was achieved by moving the correlation point along the fiber, one by one. In the work presented below, we present a new approach to measure the SBS interaction, which allows centimeter resolution over several kilometers. The approach is based on the B-OCDA method, but it extends the periodicity in order to increase the separation between nearby correlation points. To achieve this, we modulate the two waves by a binary random phase mask. We show that the resolution depends on the modulation frequency, while the distance between the correlation points depends on the entire length of the mask.

We expect to create long random sequences easily, so the SBS measurement can be extended to several kilometers.
It is possible to increase the distance between correlation points at will and thus remove the basic range limit of B-OCDA.

Analytical, numerical analysis and laboratory experiments show that it is indeed possible to locate correlation points at will along an optical fiber and thus allow access to arbitrary points.

First experiment shows a full scan of a fiber 40 meters long with a spatial resolution of a centimeter. A 5 centimeter-long piece of the fiber at the end of the fiber was well identified by the scan. Uncertainty in the measurement is equivalent to ±0.5 °C or a deformation of ±10 microstrains.

4,000 correlation points during the experiment were the highest points recorded by the system B-OCDA.

By reducing the field of acoustics to discrete points, one can implement dynamic grids on optical fibers. The grids are written by a pair of symbols modulated by a phase code, along one of the two axes. Under the correct frequency, the symbols modulated along the vertical axis may be reflected backward by the grid. The analysis of the reflected symbols allows for an analysis of the grid itself, with reflectivity, spatial size, width, and reflections outside the correlation area intended.

We found that in order to maintain the relationship between the signal to noise ratio at a reasonable level, one must reduce the unwanted reflections. Using special codes having a correlation advantage, called "Golomb Mosaic" codes, indeed reduced the unwanted reflections by a factor of 3-4. Dynamic grids have been successfully used in a demonstration of the time-continuous communication delay of a symbol at a rate of 1 Gb/s, up to 10 nanoseconds. The signal-to-noise ratio of the symbol was high enough to maintain an open eye diagram.

The increase in the measurement range of B-OCDA encoded phase poses a challenge for two reasons.

Primarily, noise is standing in the grid's reflection which occurs along the fiber.

Second, the method requires a continuous sample of the fiber at the end of the point, which leads to a long measurement time. A new method that combines the time domain and the correlation domain simplifies both domains. In this method, a pair of symbols is encoded by a short phase code creating an interaction with a large number of correlation points. In addition, the amplitude of the symbol is modulated by isolated pulses that create a separation in time between interaction of SBS in different points. The increase in the beam from one point to another is identified by a separation in the time domain.

The technical system combines the high spatial resolution and the measurement range of B-OCDA encoded phase, with a reduction in the measurement time by access to a large number of points using a single pulse. The experimental work relies on this technique a fiber of 1,600 meters long with a spatial resolution of 2 cm.

The measured range improvement is due to an enhancement of the measurement technique. This technique is based on two layers of encoding, phase, and amplitude.

The signal is acquired and processed using a short sequence of phase correlation along the signal. Additionally, the signal is processed using a sequence of amplitude in a slow rate. A small signal of the amplitude sequence on the signal is a “stamped” copy of the sequence of amplitude.

After the matched processing, the ratio of the signal to the noise received for the amplitude at the correlation points is lower than that of a single pulse, and it is actually equivalent to that received after processing on hundreds of pulses. Therefore, the time required to complete the survey decreases, enabling the measurement of more points.

The system is based on detecting the material of the fiber at a length of 8.8 km at a spatial resolution of 2 cm. Every 440,000 points are examined by 211 surveys for each frequency range.

A section of fiber of length 7 cm at the end of the fiber is identified with success. The number of points measured in this experiment is the highest achieved using a sensor with the same physical properties at the time of writing.

Finally, we present a sensor fiber based on light-matter interactions enabling the measurement of liquids outside the cover (cladding) of a standard optical fiber, without structural changes. The measurement is based on acoustical waves front scattering by acoustic modes that are supported by the optical fiber.

The acoustic waves stimulate a signal, which is detected by Sagnac interferometers. The frequencies and the measured amplitudes are suitable for quantitative analysis.

The acoustic reflections at the boundary between the fiber material and the surrounding medium are calculated based on the lifetimes of sound in different materials. The measurements were validated in terms of successful measurements of water solutions of 0%, 4%, 8%, 12%, and 16%.
מדידת אינטרפרומטרית של גלי אקוסטיים 5.3.3
шение ניסיוניות 5.4
מערך ניסוי 5.4.1
шение: סיב חשוים במים מזוקקים, אתנול ואוויר 5.4.2
ション: סיב בתמיסות מלח במכים מזוקקים 5.4.3
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טישה מבוזרת בסיבים ע”, פיזור בריליאנט מקודד פאה 6.1.1
ריריג קדמיו ע”, בריליאנט מקודד פאה 6.1.2
טישה נוזלים מצוקים לסיבים טנטורטי 6.1.3
הרבבות עתידיות 6.2
abantוס על פליטה ספונטנית מוגברת B-OCDA 6.2.1
תפונות מעבר ב- B-OCDA 6.2.2
שימוון בזוזו פוליסים ב- B-OCDA 6.2.3
מחبرا של פיזור בריליאנט ע", גלי אקוסטיים בסיום באבל ליבש מזרחי 6.2.4
הנה – קודי גולומב
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חיישני סיב-אופטימי על בסיס פיזור בריליואן
קדמי ואחורי.

希버 לשמ קבלת התואר "דוקטור לפילוסופיה"

מאテン: יאיר אנטמן
הפקולטה להנדסה

הוגש לסניאט של אוניברסיטת בר-אילן

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ינס, תשע"ז